立信会计学院

数统系

微积分

习题解答

Jinsw

习题7～1：P260．5

$$M(-4,3,-5)$$

$$M_x(-4,0,0) \qquad M_y(0,3,0) \qquad M_z(0,0,-5)$$

$$|MM_x| = \sqrt{[-4-(-4)]^2+(3-0)^2+(-5-0)^2}$$

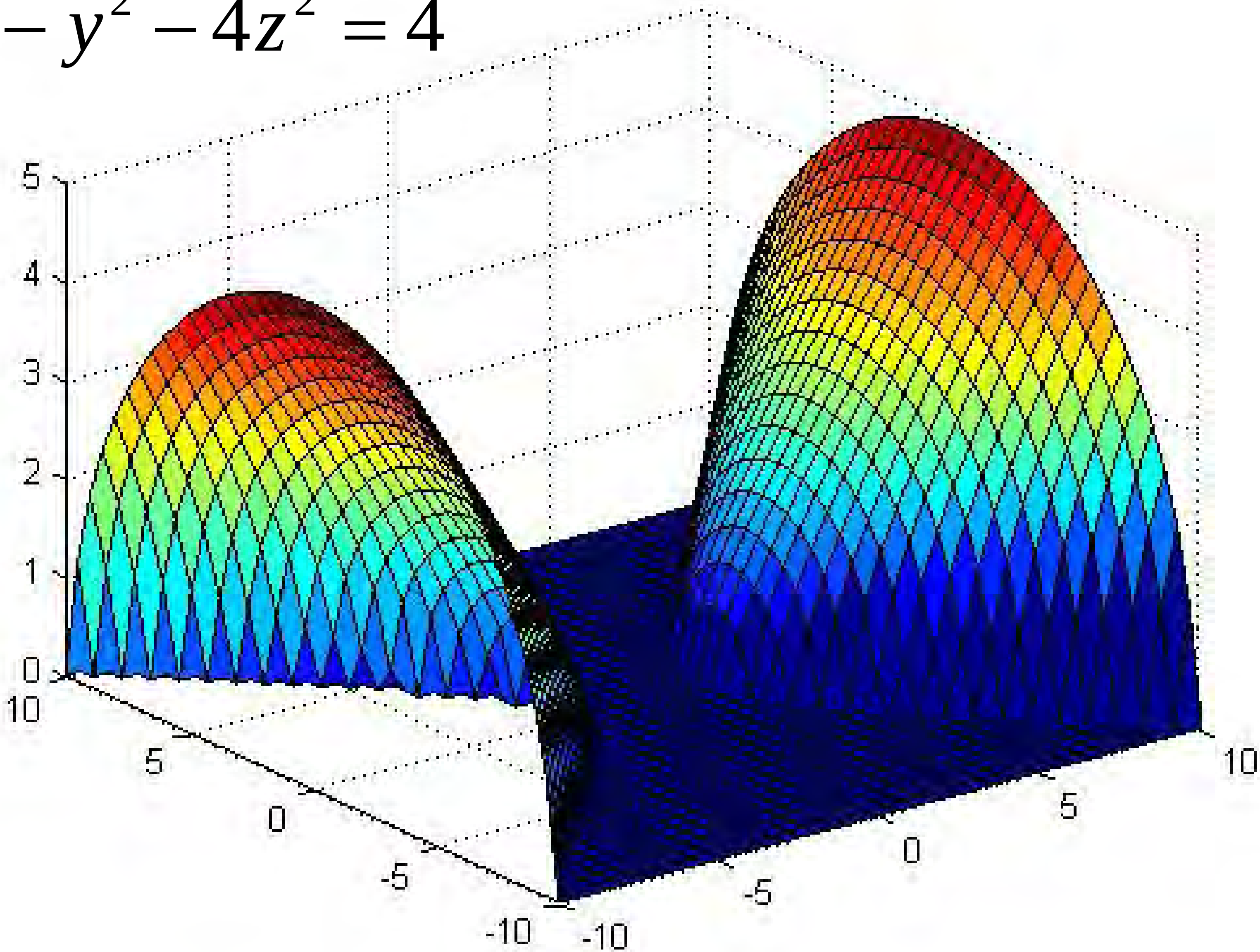
$$= \sqrt{34}$$

$$|MM_y| = \sqrt{(-4-0)^2+(-5-0)^2} = \sqrt{41}$$

$$|MM_z| = \sqrt{(-4-0)^2+(3-0)^2} = 5$$

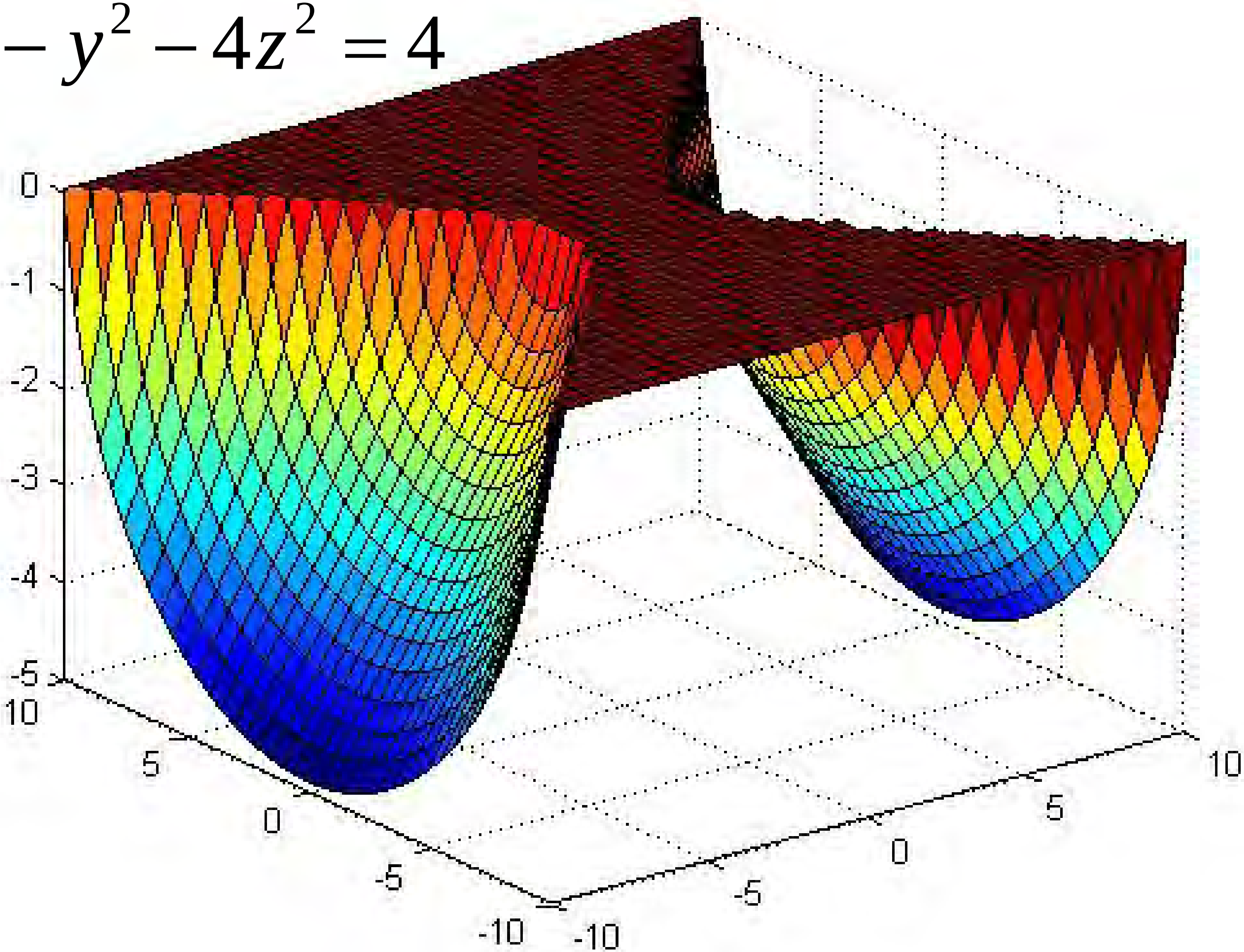
习题7～4：Ｐ２７０．１（２）　—

$$x^2 - y^2 - 4z^2 = 4$$



习题7～4：P270. 1（2） 二

$$x^2 - y^2 - 4z^2 = 4$$



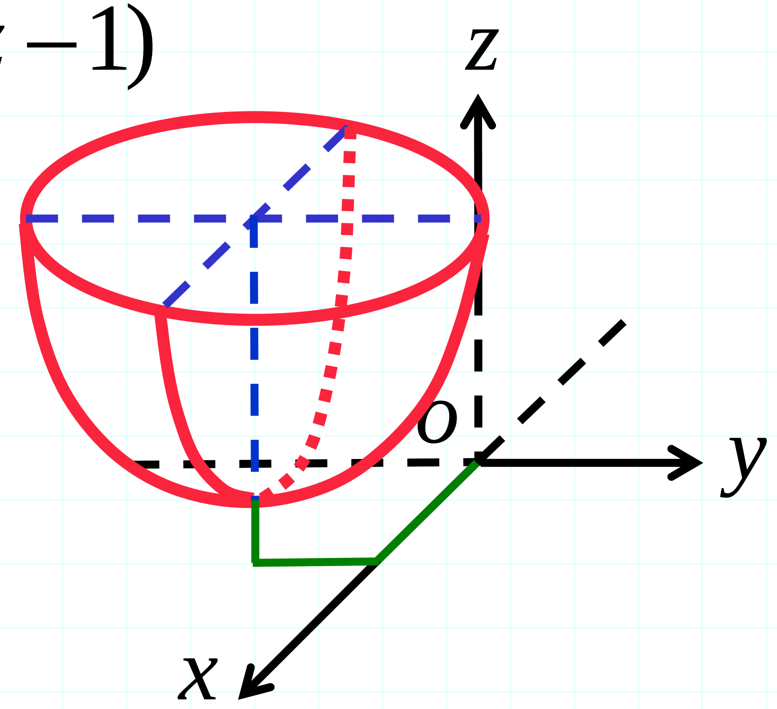
总习题七：Ｐ２７０．１

$$|z|=\sqrt{(x-4)^2+(y+2)^2+(z-2)^2}$$

$$x^2+y^2-6x+4y-4z+17=0$$

$$(x-3)^2+(y+2)^2=4(z-1)$$

旋转抛物面



习题7～2：P286.5

$$\vec{a}=\{a_1,a_2,a_3\}\quad \vec{b}=\{b_1,b_2,b_3\}$$

$$\vec{a}\cdot\vec{b}=|\vec{a}|\cdot|\vec{b}|\cdot\cos\theta$$

$$|\vec{a}\cdot\vec{b}|\le|\vec{a}|\cdot|\vec{b}|$$

$$|a_1\cdot b_1+a_2\cdot b_2+a_3\cdot b_3|$$

$$\le\sqrt{a_1^2+a_2^2+a_3^2}\cdot\sqrt{b_1^2+b_2^2+b_3^2}$$

习题7～4：P296．1（4）

$(1,1,-1)\quad(-2,-2,2)\quad(1,-1,2)$

$$\begin{vmatrix} x-1 & y-1 & z+1 \\ -2-1 & -2-1 & 2+1 \\ 1-1 & -1-1 & 2+1 \end{vmatrix}=0$$

$$x-3y-2z=0$$

习题7～4：P297. 15

$$\begin{cases} x+2y+5=0 \\ 2y-z-4=0 \end{cases} \qquad \begin{cases} y=0 \\ x+2z+4=0 \end{cases}$$

$$l_1: \frac{x+5}{-2}=\frac{y}{1}=\frac{z+4}{2} \qquad l_2: \frac{x}{2}=\frac{y}{0}=\frac{z+2}{-1}$$

$$\vec{s}_{公}=\vec{s}_1\times\vec{s}_2=\{-1,2,-2\}$$

π_1：含 l_1 且平行于 $\vec{s}_{公}$
$$\begin{vmatrix} x+5 & y & z+4 \\ -2 & 1 & 2 \\ -1 & 2 & -2 \end{vmatrix}=0$$

$$\Rightarrow 2x+2y+z+14=0$$

习题7～4：P297.15（续）

π_1：含l_1且平行于$\vec{s}_{公}$ $\Rightarrow 2x+2y+z+14=0$

π_2：含l_2且平行于$\vec{s}_{公}$ $\begin{vmatrix} x & y & z+2 \\ 2 & 0 & -1 \\ -1 & 2 & -2 \end{vmatrix}=0$

$\Rightarrow 2x+5y+4z+8=0$

π_1与π_1的交线：$\begin{cases} 2x+2y+z+14=0 \\ 2x+5y+4z+8=0 \end{cases}$

$\Rightarrow \dfrac{x+9}{1}=\dfrac{y-2}{-2}=\dfrac{z}{2}$ ——公垂线

习题７～５：Ｐ３０５．１

$$\sqrt{(x-5)^2+(y-4)^2+z^2}$$

$$=2\cdot\sqrt{(x+4)^2+(y-3)^2+(z-4)^2}$$

$$(x+7)^2+(y-\frac{8}{3})^2+(z-\frac{16}{3})^2=\frac{392}{9}$$

球面

习题８～１：Ｐ３０２．１（１）

$$f(x,y)=x^2-y^2$$

$$f\left(x+y,\frac{y}{x}\right)=(x+y)^2-\left(\frac{y}{x}\right)^2$$

习题8～1：P302．1（2）

$$f(x+y,\frac{y}{x})=x^2-y^2$$

令 $x+y=u,\frac{y}{x}=v \Rightarrow x=\frac{u}{1+v},y=\frac{uv}{1+v}$

$$f(u,v)=\left(\frac{u}{1+v}\right)^2-\left(\frac{uv}{1+v}\right)^2=\frac{u^2\cdot(1-v)}{1+v}$$

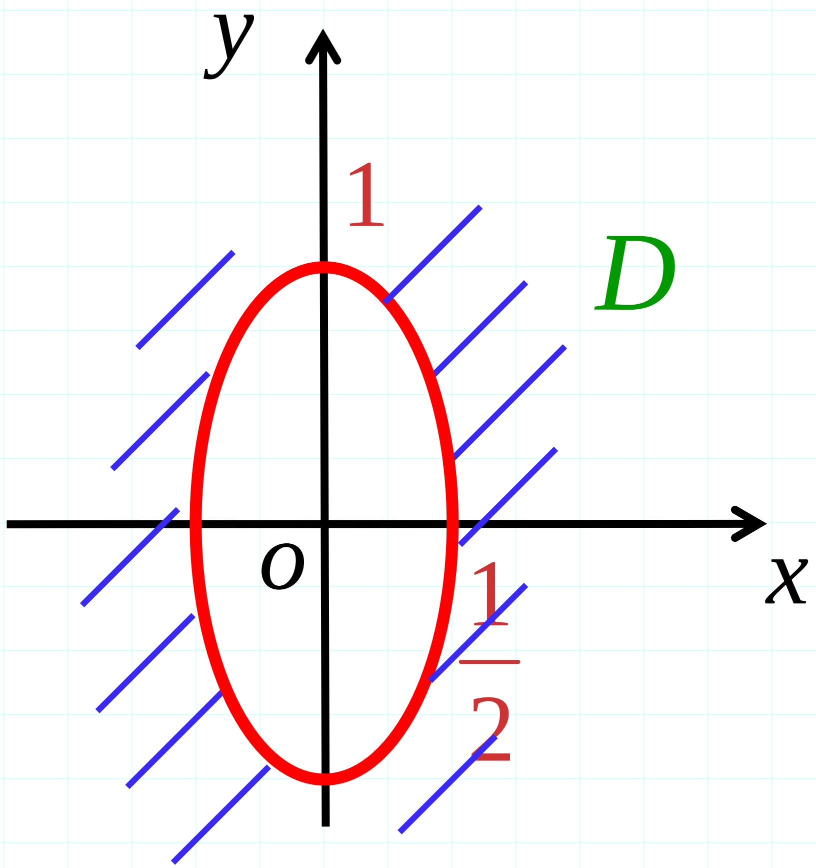
$$f(x,y)=\frac{x^2\cdot(1-y)}{1+y}$$

习题8～1：P302．2（1）

$$z=\sqrt{4x^2+y^2-1}$$

$$x^2+\frac{y^2}{4}\geq 1$$

椭圆外

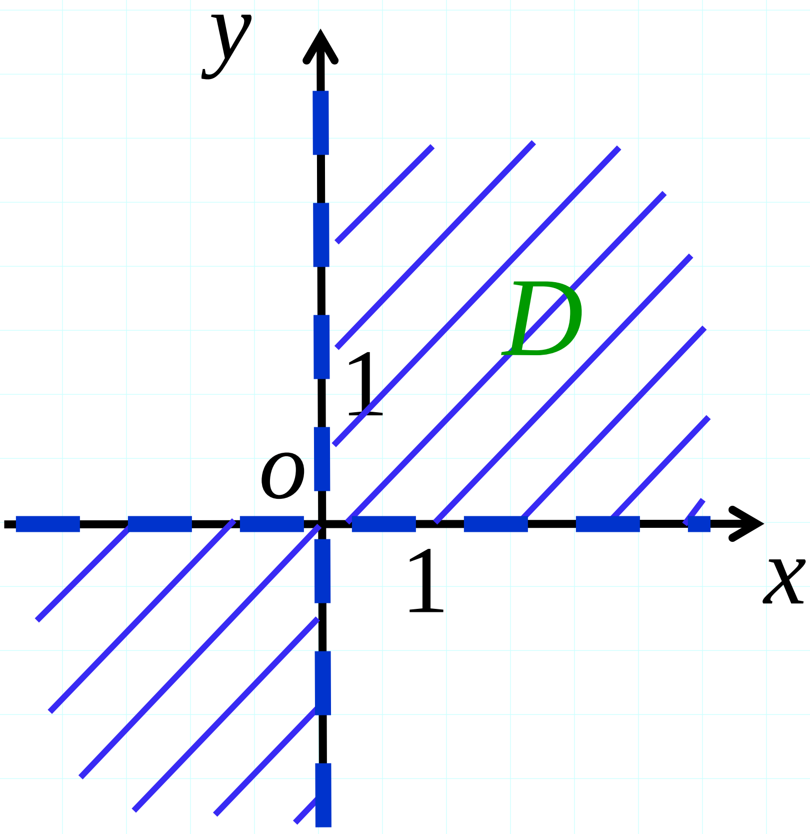


习题8～1：P302. 2（2）

$$z=\ln(x\cdot y)$$

$$x\cdot y>0$$

Ⅰ、Ⅲ象限

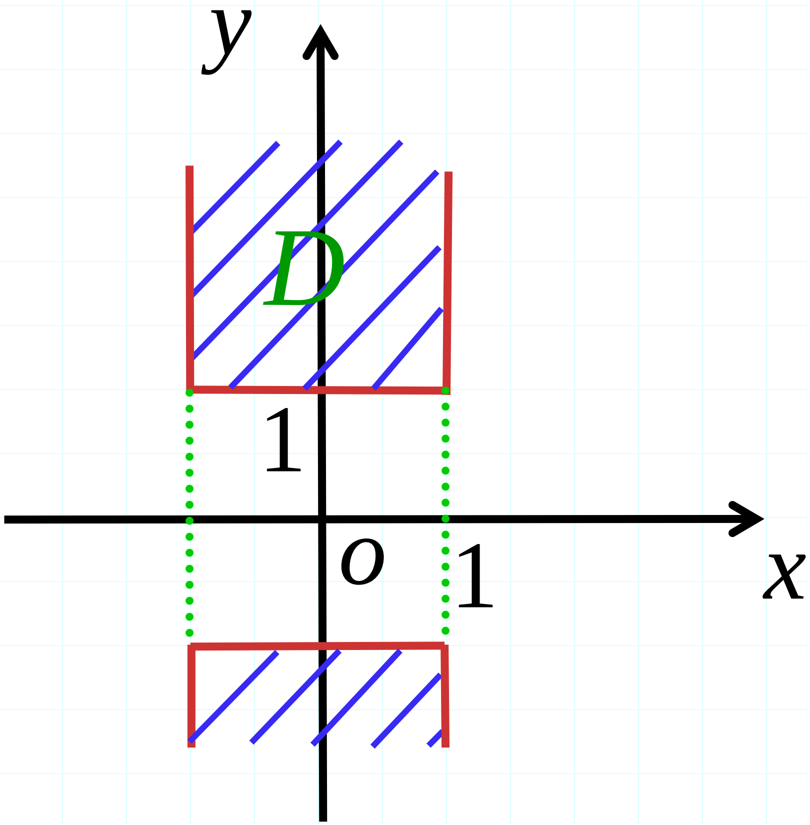


习题8～1：P302. 2（3）

$$z=\sqrt{1-x^2}+\sqrt{y^2-1}$$

$$|x|\le 1, |y|\ge 1$$

带状



习题8～1：P302．2（4）

$$z=\ln(1-|x|-|y|)$$

$$1-|x|-|y|>0 \quad |x|+|y|<1$$

Ⅰ $x\geq 0 \quad x+|y|<1$

$|y|<1-x \Rightarrow x<1$

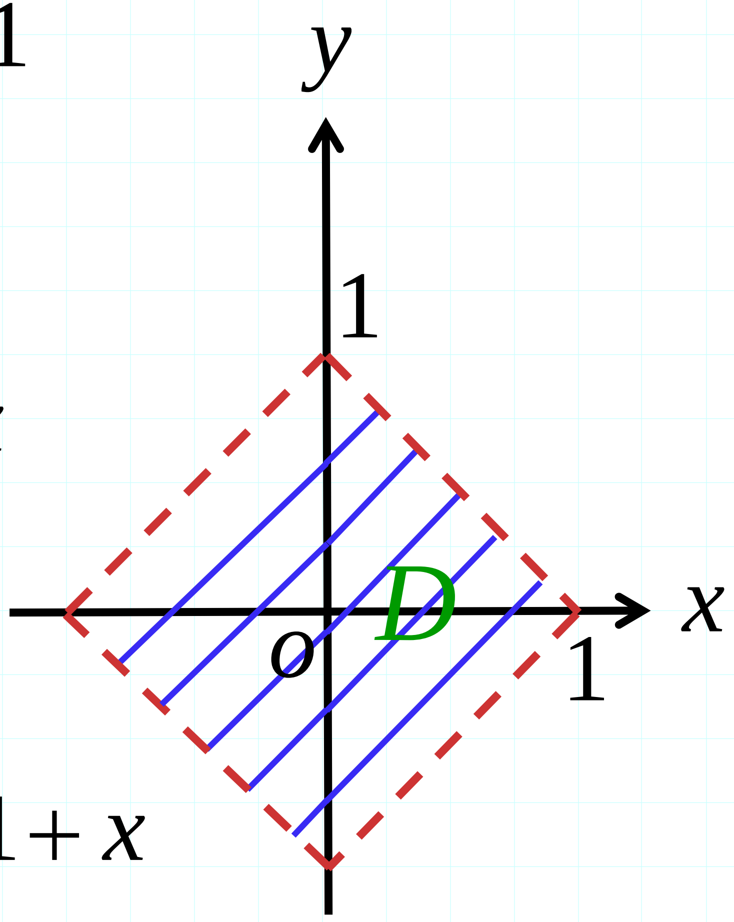
即 $0\leq x<1, x-1<y<1-x$

Ⅱ $x<0 \quad -x+|y|<1$

$|y|<1+x \Rightarrow x>-1$

即 $-1<x<0, -x-1<y<1+x$

菱形（正方形）



习题８～１：Ｐ３１８．３（１）

$$\lim_{(x,y)\to(0,0)} \frac{x^2\cdot y}{x^3-y^3}$$ 极限不存在

$y=2x$

$$\lim_{(x,y)\to(0,0)} \frac{x^2\cdot y}{x^3-y^3}=\lim_{x\to 0}\frac{x^2\cdot 2x}{x^3-(2x)^3}=-\frac{2}{7}$$

$y=3x$

$$\lim_{(x,y)\to(0,0)} \frac{x^2\cdot y}{x^3-y^3}=\lim_{x\to 0}\frac{x^2\cdot 3x}{x^3-(3x)^3}=-\frac{3}{26}$$

习题8～1：P318．3（2）

$$\lim_{(x,y)\to(0,0)} \frac{x\cdot y}{x+y}$$ 极限不存在

$$y = x$$

$$\lim_{(x,y)\to(0,0)} \frac{x\cdot y}{x+y} = \lim_{x\to 0} \frac{x\cdot x}{x+x} = 0$$

$$y = x^2 - x$$

$$\lim_{(x,y)\to(0,0)} \frac{x\cdot y}{x+y} = \lim_{x\to 0} \frac{x\cdot(x^2-x)}{(x^2-x)+x}$$

$$= \lim_{x\to 0} \frac{x^3-x^2}{x^2} = \lim_{x\to 0}(x-1) = -1$$

习题8～1：P318．4（1）

$$\lim_{(x,y)\to(1,3)} \frac{x\cdot y}{\sqrt{x\cdot y+1}-1}$$

$$=\lim_{(x,y)\to(1,3)} \frac{x\cdot y(\sqrt{x\cdot y+1}+1)}{(\sqrt{x\cdot y+1}-1)(\sqrt{x\cdot y+1}+1)}$$

$$=\lim_{(x,y)\to(1,3)} (\sqrt{x\cdot y+1}+1)$$

$$=3$$

习题８～１：Ｐ３１８．４（２）

$$\lim_{(x,y)\to(0,0)} \frac{2-\sqrt{x\cdot y+4}}{x\cdot y}$$

$$= \lim_{(x,y)\to(0,0)} \frac{(2-\sqrt{x\cdot y+4})(2+\sqrt{x\cdot y+4})}{x\cdot y\cdot(2+\sqrt{x\cdot y+4})}$$

$$= \lim_{(x,y)\to(0,0)} \frac{4-(x\cdot y+4)}{x\cdot y\cdot(2+\sqrt{x\cdot y+4})}$$

$$= -\frac{1}{4}$$

习题8～1：P318．4（3）

$$\lim_{(x,y)\to(0,0)}\left(x\cdot\sin\frac{1}{y}+y\cdot\sin\frac{1}{x}\right)=0$$

$$0\le\left|x\cdot\sin\frac{1}{y}+y\cdot\sin\frac{1}{x}\right|\le\left|x\cdot\sin\frac{1}{y}\right|+\left|y\cdot\sin\frac{1}{x}\right|$$

$$=|x|\cdot\left|\sin\frac{1}{y}\right|+|y|\cdot\left|\sin\frac{1}{x}\right|\le|x|+|y|$$

$$\le\sqrt{x^2+y^2}+|y|\le2\cdot\sqrt{x^2+y^2}\to0$$

习题8～1：P318．4（4）

$$\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2}=0$$

$$0\le\left|\frac{x^3+y^3}{x^2+y^2}\right|=\frac{|x+y|\cdot|x^2-x\cdot y+y^2|}{x^2+y^2}$$

$$\le\frac{|x+y|\cdot(|x^2+y^2|+|x\cdot y|)}{x^2+y^2}$$

$$\le\frac{|x+y|\cdot(|x^2+y^2|+\frac{x^2+y^2}{2})}{x^2+y^2}=\frac{3}{2}|x+y|\to 0$$

习题８～１：Ｐ３１８．６

$$\forall P \in \Omega$$

$$d = |PM_0| = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$$

$r = \sqrt{x^2 + y^2 + z^2}$ 在 R^3 中连续 $\Rightarrow$

$d = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$ 在 Ω 中连续，Ω 为有界闭区域，d 在 Ω 上必有最值。

证毕

习题8～2：P311.1（1）

$$z=\frac{3}{y^{2}}-\frac{1}{\sqrt[3]{x}}+\ln 5$$

$$z_{x}=\frac{1}{3}x^{-\frac{4}{3}}$$

$$z_{y}=-6y^{-3}$$

习题8～2：P311. 1（2）

$$z=\sqrt{\ln(xy)}$$

$$z_x=\frac{1}{2\sqrt{\ln(xy)}}\cdot\frac{1}{xy}\cdot y$$

$$=\frac{1}{2x\sqrt{\ln(xy)}}$$

$$z_y=\frac{1}{2y\sqrt{\ln(xy)}}$$

习题8～2：P311.1（3）

$$S = \frac{u+v}{u-v}$$

$$S_u = \frac{-2v}{(u-v)^2}$$

$$S_v = \frac{2u}{(u-v)^2}$$

习题８～２：Ｐ３１１．１（４）

$$z=\ln\tan\frac{x}{y}$$

$$z_x=\frac{1}{\tan\frac{x}{y}}\cdot\sec^2\frac{x}{y}\cdot\frac{1}{y}=\frac{2}{y}\cdot\csc\frac{2x}{y}$$

$$z_x=\frac{-2x}{y^2}\cdot\csc\frac{2x}{y}$$

习题8～2：P311．1（5）

$$u=\sin\frac{x}{y}\cdot\cos\frac{y}{x}+z$$

$$u_x=\frac{1}{y}\cos\frac{x}{y}\cdot\cos\frac{y}{x}+\frac{y}{x^2}\sin\frac{x}{y}\cdot\sin\frac{y}{x}$$

$$u_y=-\frac{x}{y^2}\cos\frac{x}{y}\cdot\cos\frac{y}{x}-\frac{1}{x}\sin\frac{x}{y}\cdot\sin\frac{y}{x}$$

$$u_z=1$$

习题8～2：P311.1（6）

$$u = x^{\frac{y}{z}}$$

$$u_x = \frac{y}{z} x^{\frac{y}{z}-1}$$

$$u_y = x^{\frac{y}{z}} \cdot \ln x \cdot \frac{1}{z}$$

$$u_z = x^{\frac{y}{z}} \cdot \ln x \cdot (\frac{-y}{z^2})$$

习题8～2：P311.1（7）

$$z=(1+x\cdot y)^{y}$$ ………… 对 y 为幂指函数

$$z_x=y\cdot(1+x\cdot y)^{y-1}\cdot y$$

$$=y^2\cdot(1+x\cdot y)^{y-1}$$

$$\ln z=\ln(1+x\cdot y)^{y}=y\cdot\ln(1+x\cdot y)$$

$$z_y=(1+x\cdot y)^{y}\cdot[\ln(1+x\cdot y)+\frac{x\cdot y}{1+x\cdot y}]$$

习题８～２：Ｐ３１１．１（８）

$$f(\rho,\varphi,t)=\rho\cdot e^{t\cdot\varphi}+e^{-\varphi}+t$$

$$f_\rho(\rho,\varphi,t)=e^{t\cdot\varphi}$$

$$f_\varphi(\rho,\varphi,t)=\rho\cdot e^{t\cdot\varphi}\cdot t-e^{-\varphi}$$

$$f_t(\rho,\varphi,t)=\rho\cdot e^{t\cdot\varphi}+1$$

习题８～２：Ｐ３１２．５（１）

$$z = x^{2y}$$

$$z_x = 2y \cdot x^{2y-1}$$

$$z_{xx} = (2y) \cdot (2y-1) \cdot x^{2y-2}$$

$$z_y = x^{2y} \cdot \ln x \cdot 2$$

$$z_{yy} = x^{2y} \cdot (\ln x \cdot 2)^2 = 4x^{2y} \cdot \ln^2 x$$

$$z_{xy} = 2x^{2y-1} + 2y \cdot x^{2y-1} \cdot \ln x \cdot 2$$

$$= 2x^{2y-1}(1 + 2\ln x)$$

习题 8～2：P 3 1 2. 5（2）

$$z = \arctan\frac{y}{x}$$

$$z_x = \frac{1}{1+\left(\frac{y}{x}\right)^2}\left(-\frac{y}{x^2}\right) = -\frac{y}{x^2+y^2}$$

$$z_{xx} = -\frac{y(-2x)}{(x^2+y^2)^2} = \frac{2xy}{(x^2+y^2)^2}$$

习题８～２：Ｐ３１２．５（２）续

$$z = \arctan\frac{y}{x}$$

$$z_y = \frac{1}{1+\left(\frac{y}{x}\right)^2}\left(\frac{1}{x}\right) = \frac{x}{x^2+y^2}$$

$$z_{yy} = \frac{x(-2y)}{(x^2+y^2)^2} = \frac{-2xy}{(x^2+y^2)^2}$$

$$z_{xy} = -\frac{(x^2+y^2)-y(2y)}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$$

习题8～2：P312．7

$$z = 2\cos^2(x - \frac{t}{2})$$

$$z_x = 2 \cdot 2 \cdot \cos(x - \frac{t}{2}) \cdot [-\sin(x - \frac{t}{2})] = -2\sin[2(x - \frac{t}{2})]$$

$$z_{xt} = -2 \cdot \cos[2(x - \frac{t}{2})] \cdot 2 \cdot (-\frac{1}{2}) = 2 \cdot \cos[2(x - \frac{t}{2})]$$

$$z_t = 2 \cdot 2 \cdot \cos(x - \frac{t}{2}) \cdot [-\sin(x - \frac{t}{2})] \cdot (-\frac{1}{2}) = \sin[2(x - \frac{t}{2})]$$

$$z_{tt} = \cos[2(x - \frac{t}{2})] \cdot 2 \cdot (-\frac{1}{2}) = -\cos[2(x - \frac{t}{2})]$$

$$2 \cdot z_{tt} + z_{xt} = 0$$

习题 8～2：P 3 2 7. 8

$$P_X = 1000 - 5Q_X \Rightarrow Q_X = 0.2\cdot(1000 - P_X)$$

$$P_Y = 1600 - 4Q_Y \Rightarrow Q_Y = 0.25\cdot(1600 - P_Y)$$

$$\eta_X = \frac{\partial Q_X}{\partial P_X}\cdot\frac{P_X}{Q_X} = -0.2\cdot\frac{P_X}{0.2\cdot(1000 - P_X)} = \frac{P_X}{P_X - 1000}$$

$$\eta_Y = \frac{\partial Q_Y}{\partial P_Y}\cdot\frac{P_Y}{Q_Y} = -0.25\cdot\frac{P_Y}{0.25\cdot(1600 - P_Y)} = \frac{P_Y}{P_Y - 1600}$$

$$Q_{X_0} = 100 \Rightarrow P_{X_0} = 500 \quad \eta_X\Big|_{P_{X_0}=100} = \frac{500}{500 - 1000} = -1$$

$$Q_{Y_0} = 250 \Rightarrow P_{Y_0} = 600 \quad \eta_Y\Big|_{P_{Y_0}=250} = \frac{600}{600 - 1600} = -0.6$$

$$Q_{Y^*} = 300 \Rightarrow P_{Y^*} = 400 \quad \frac{\Delta Q_X}{Q_X}\Big/\frac{\Delta P_Y}{P_Y} = \frac{75 - 100}{100}\Big/\frac{400 - 600}{600} = 0.75$$

习题８～３：Ｐ３１７．１（１）

$$z = 3x \cdot e^{-y} - 2\sqrt{x} + \ln 5$$

$$z_x = 3e^{-y} - \frac{1}{\sqrt{x}}$$

$$z_y = -3x \cdot e^{-y}$$

$$dz = (3e^{-y} - \frac{1}{\sqrt{x}})dx - 3x \cdot e^{-y} dy$$

习题 8～3：P317. 1（2）

$$z = e^{\frac{y}{x}}$$

$$z_x = e^{\frac{y}{x}} \cdot \left(-\frac{y}{x^2} \right)$$

$$z_y = e^{\frac{y}{x}} \cdot \frac{1}{x}$$

$$dz = e^{\frac{y}{x}} \cdot \left(-\frac{y}{x^2} dx + \frac{1}{x} dy \right)$$

习题８～３：Ｐ３１７．１（３）

$$u = y^{x\cdot z}$$

$$u_x = y^{x\cdot z} \cdot \ln y \cdot z$$

$$u_y = x \cdot z \cdot y^{x\cdot z-1}$$

$$u_z = y^{x\cdot z} \cdot \ln y \cdot x$$

$$du = y^{x\cdot z} \cdot \left(z \cdot \ln y dx + \frac{x \cdot z}{y} dy + x \cdot \ln y dz \right)$$

习题8～3：P317. 2

$$z=\ln(1+x^2+y^2)\qquad x=1, y=2$$

$$dz=\frac{2x\cdot dx+2y\cdot dy}{1+x^2+y^2}$$

$$dz\Big|_{x=1,y=2}=\frac{1}{3}dx+\frac{2}{3}dy$$

习题８～３：Ｐ３１７．３

$$z = e^{x \cdot y}$$

$$x = 1, y = 1 \quad \Delta x = 0.1, \Delta y = -0.2$$

$$dz = e^{x \cdot y} \cdot (y \cdot dx + x \cdot dy)$$

$$dz \Big|_{\substack{x=1, y=1 \\ \Delta x = 0.1, \Delta y = -0.2}} = -0.1e$$

习题8～3：P333.4（1）

$$(1.04)^{2.02} \qquad z = x^y$$

$$x_0 = 1, y_0 = 2 \quad \Delta x = 0.04, \Delta y = 0.02$$

$$dz = x^y \cdot (\frac{y}{x} \cdot dx + \ln x \cdot dy)$$

$$(1.04)^{2.02} \approx 1^2 + 1^2 \cdot (\frac{2}{1} \cdot 0.04 + \ln 1 \cdot 0.02)$$

$$= 1.08$$

$$(1.04)^{2.02} = 1.082448755$$

习题８～３：Ｐ３３３．４（２）

$$\sqrt{(1.02)^3+(1.97)^3} \qquad z=\sqrt{x^3+y^3}$$

$$x_0=1,\, y_0=2 \qquad \Delta x=0.02,\, \Delta y=-0.03$$

$$dz=\frac{3x^2\cdot dx+3y^2\cdot dy}{2\sqrt{x^3+y^3}}$$

$$\sqrt{(1.02)^3+(1.97)^3}$$

$$\approx\sqrt{1^3+2^3}+\frac{3\cdot 1^2\cdot 0.02+3\cdot 2^2\cdot(-0.03)}{2\sqrt{1^3+2^3}}=2.95$$

$$\sqrt{(1.02)^3+(1.97)^3}=2.950691614$$

习题8～3：P333．5

$$x_0 = 6, y_0 = 8 \quad \Delta x = 0.002, \Delta y = -0.005$$

$$z = \sqrt{x^2 + y^2} \qquad dz = \frac{x \cdot dx + y \cdot dy}{\sqrt{x^3 + y^3}}$$

$$\Delta z \approx \frac{6 \cdot 0.002 + 8 \cdot (-0.005)}{\sqrt{6^2 + 8^2}} = -2.8 \quad (mm)$$

$$s = x \cdot y \qquad ds = y \cdot dx + x \cdot dy$$

$$\Delta s \approx 8 \times 0.002 + 6 \times (-0.005) = -14 \quad (mm)^2$$

$$\Delta z = -2.7989418 \qquad \Delta s = -14.01$$

习题8～3：P333.6

$$V = \pi \cdot r^2 \cdot h$$

$$r_0 = 4, h_0 = 20 \quad \Delta r = 0.1, \Delta h = 0.1$$

$$dV = \pi \cdot 2r \cdot h \cdot dr + \pi \cdot r^2 \cdot dh$$

$$\Delta V \approx \pi \cdot (2 \times 4 \times 20 \times 0.1 + 4^2 \times 0.1)$$

$$= 17.6\pi \quad (cm)^3$$

$$\Delta V = 17.881\pi$$

习题8～4：P323．1（1）

$$z=\frac{v}{u},u=\ln x,v=e^{x}$$

$$\frac{\partial z}{\partial u}=\frac{v}{u^{2}}\quad \frac{\partial z}{\partial v}=\frac{1}{u}\quad \frac{du}{dx}=\frac{1}{x}\quad \frac{dv}{dx}=e^{x}$$

$$\frac{dz}{dx}=-\frac{e^{x}\cdot\frac{1}{x}}{(\ln x)^{2}}+\frac{e^{x}}{\ln x}=-\frac{e^{x}\cdot(x\cdot\ln x-1)}{x\cdot(\ln x)^{2}}$$

习题 8～4：P 3 2 3．1（2）

$$z=\arctan(x-y), x=3t, y=4t^3$$

$$\frac{\partial z}{\partial x}=\frac{1}{1+(x-y)^2} \quad \frac{\partial z}{\partial y}=\frac{-1}{1+(x-y)^2}$$

$$\frac{dx}{dt}=3 \quad \frac{dy}{dt}=12t^2$$

$$\frac{dz}{dt}=\frac{3-12t^2}{1+(x-y)^2}=\frac{3-12t^2}{1+(3t-4t^3)^2}$$

习题8～4：P323. 1（3）

$$z = x \cdot y + y \cdot t, y = 2x, t = \sin x$$

$$\frac{\partial z}{\partial y} = x + t \qquad \frac{\partial z}{\partial t} = y \qquad \frac{\partial f}{\partial x} = y$$

$$\frac{dy}{dx} = 2^x \cdot \ln 2 \qquad \frac{dt}{dx} = \cos x$$

$$\frac{dz}{dx} = \frac{\partial f}{\partial x} + \frac{\partial z}{\partial y}\frac{dy}{dx} + \frac{\partial z}{\partial t}\frac{dt}{dx}$$

$$= y + (x + t) \cdot 2^x \cdot \ln 2 + y \cdot \cos x$$

$$= 2^x \cdot (1 + x \cdot \ln 2 + \sin x \cdot \ln 2 + \cos x)$$

习题8～4：P323.2（1）

$$z = u \cdot e^{\frac{u}{v}}, u = x^2 + y^2, v = x \cdot y$$

$$\frac{\partial z}{\partial u} = e^{\frac{u}{v}} + u \cdot e^{\frac{u}{v}} \cdot \frac{1}{v} \qquad \frac{\partial z}{\partial v} = u \cdot e^{\frac{u}{v}} \cdot \left(\frac{-u}{v^2} \right)$$

$$\frac{\partial u}{\partial x} = 2x \qquad \frac{\partial u}{\partial y} = 2y \qquad \frac{\partial v}{\partial x} = y \qquad \frac{\partial v}{\partial y} = x$$

$$\frac{\partial z}{\partial x} = e^{\frac{u}{v}} \cdot \left(1 + \frac{u}{v} \right) \cdot 2x + e^{\frac{u}{v}} \cdot \frac{u^2}{v^2} \cdot y$$

$$= e^{\frac{x^2+y^2}{x \cdot y}} \cdot \left[2x + \frac{2(x^2+y^2)}{y} - \frac{(x^2+y^2)^2}{x^2 \cdot y} \right]$$

习题8～4：P323．2（1）续

$$z = u \cdot e^{\frac{u}{v}}, u = x^2 + y^2, v = x \cdot y$$

$$\frac{\partial z}{\partial u} = e^{\frac{u}{v}} + u \cdot e^{\frac{u}{v}} \cdot \frac{1}{v} \qquad \frac{\partial z}{\partial v} = u \cdot e^{\frac{u}{v}} \cdot \left(\frac{-u}{v^2} \right)$$

$$\frac{\partial u}{\partial x} = 2x \qquad \frac{\partial u}{\partial y} = 2y \qquad \frac{\partial v}{\partial x} = y \qquad \frac{\partial v}{\partial y} = x$$

$$\frac{\partial z}{\partial y} = e^{\frac{u}{v}} \cdot \left(1 + \frac{u}{v} \right) \cdot 2y - e^{\frac{u}{v}} \cdot \frac{u^2}{v^2} \cdot x$$

$$= e^{\frac{x^2+y^2}{x \cdot y}} \cdot \left[2y + \frac{2(x^2+y^2)}{x} - \frac{(x^2+y^2)^2}{x \cdot y^2} \right]$$

习题8～4：P323．2（2）

$$z = x^2 \cdot \ln y, x = \frac{u}{v}, y = 3u - 2v$$

$$z_x = 2x \cdot \ln y \quad z_y = \frac{x^2}{y}$$

$$x_u = \frac{1}{v} \quad x_v = \frac{-u}{v^2} \quad y_u = 3 \quad y_v = -2$$

$$\frac{\partial z}{\partial u} = 2x \cdot \ln y \cdot \frac{1}{v} + \frac{x^2}{y} \cdot 3$$

$$= \frac{2u}{v^2} \cdot \ln(3u - 2v) + \frac{3u^2}{v^2 \cdot (3u - 2v)}$$

习题8～4：P323．2（2）续

$$z = x^2 \cdot \ln y, x = \frac{u}{v}, y = 3u - 2v$$

$$z_x = 2x \cdot \ln y \quad z_y = \frac{x^2}{y}$$

$$x_u = \frac{1}{v} \quad x_v = \frac{-u}{v^2} \quad y_u = 3 \quad y_v = -2$$

$$\frac{\partial z}{\partial v} = 2x \cdot \ln y \cdot \left(-\frac{u}{v^2}\right) + \frac{x^2}{y} \cdot (-2)$$

$$= -\frac{2u^2}{v^3} \cdot \ln(3u - 2v) + \frac{2u^2}{v^2 \cdot (3u - 2v)}$$

习题8～4：P323. 2（3）

$$z = f(x^2 - y^2, e^{x \cdot y})$$

$$u = x^2 - y^2, v = e^{x \cdot y}$$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot 2x + \frac{\partial z}{\partial v} \cdot y \cdot e^{x \cdot y}$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} \cdot (-2y) + \frac{\partial z}{\partial v} \cdot x \cdot e^{x \cdot y}$$

习题8～4：P323．2（4）

$$u = f\left(\frac{x}{y}, \frac{y}{z}\right)$$

$$\frac{\partial u}{\partial x} = f_1' \cdot \frac{1}{y}$$

$$\frac{\partial u}{\partial y} = f_1' \cdot \left(-\frac{x}{y^2}\right) + f_2' \cdot \frac{1}{z}$$

$$\frac{\partial u}{\partial z} = f_2' \cdot \left(-\frac{y}{z^2}\right)$$

习题8～4：P323．2（5）

$$u = f(x, x\cdot y, x\cdot y\cdot z)$$

$$\frac{\partial u}{\partial x} = f_1' + f_2'\cdot y + f_3'\cdot y\cdot z$$

$$\frac{\partial u}{\partial y} = f_2'\cdot x + f_3'\cdot x\cdot z$$

$$\frac{\partial u}{\partial z} = f_3'\cdot x\cdot y$$

习题 8～4：P339．3

$$z=\frac{y}{f(x^2-y^2)} \qquad u=x^2-y^2$$

$$\frac{\partial z}{\partial x}=\frac{-y\cdot f'\cdot 2x}{f^2} \qquad \frac{\partial z}{\partial y}=\frac{f-y\cdot f'\cdot(-2y)}{f^2}$$

$$\frac{1}{x}\cdot\frac{\partial z}{\partial x}+\frac{1}{y}\cdot\frac{\partial z}{\partial y}=\frac{-2y\cdot f'}{f^2}+\frac{f+y^2\cdot f'}{y\cdot f^2}$$

$$=\frac{-2y^2\cdot f'+f+y^2\cdot f'}{y\cdot f^2}=\frac{1}{y\cdot f}=\frac{z}{y^2}$$

习题8～4：P339．4

$$u = F(x, y) \quad x = r\cdot\cos\theta, y = r\cdot\sin\theta$$

$$\frac{\partial u}{\partial r} = F_x\cdot\cos\theta + F_y\cdot\sin\theta$$

$$\frac{\partial u}{\partial \theta} = F_x\cdot(-r\cdot\sin\theta) + F_y\cdot r\cdot\cos\theta$$

$$\left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{\partial u}{\partial \theta}\right)^2 = (F_x\cdot\cos\theta + F_y\cdot\sin\theta)^2 + \\ +(-F_x\cdot r\cdot\sin\theta + F_y\cdot r\cdot\cos\theta)^2 \\ = (F_x)^2 + (F_y)^2$$

习题８～４：Ｐ３２３．５（１）

$$z = \sin^2(a \cdot x + b \cdot y)$$

$$z_x = 2\sin(a \cdot x + b \cdot y) \cdot \cos(a \cdot x + b \cdot y) \cdot a$$

$$= a \cdot \sin[2(a \cdot x + b \cdot y)]$$

$$z_{xx} = 2a^2 \cos[2(a \cdot x + b \cdot y)]$$

$$z_{xy} = 2ab \cdot \cos[2(a \cdot x + b \cdot y)]$$

$$z_{yy} = 2b^2 \cos[2(a \cdot x + b \cdot y)]$$

习题8～4：P323．5（2）

$$z=\ln(y+\sqrt{x^2+y^2})$$

$$z_x=\frac{1}{y+\sqrt{x^2+y^2}}\cdot\frac{2x}{2\sqrt{x^2+y^2}}$$

$$=\frac{x\cdot(y-\sqrt{x^2+y^2})}{[y^2-(x^2+y^2)]\cdot\sqrt{x^2+y^2}}$$

$$=-\frac{y}{x\cdot\sqrt{x^2+y^2}}+\frac{1}{x}$$

$$z_{xx}=\frac{y}{x^2\cdot\sqrt{x^2+y^2}}+\frac{y}{\sqrt{(x^2+y^2)^3}}-\frac{1}{x^2}$$

习题8～4：P323.5（2）续

$$z=\ln(y+\sqrt{x^2+y^2})$$

$$z_y=\frac{1}{y+\sqrt{x^2+y^2}}\cdot\left(1+\frac{2y}{2\sqrt{x^2+y^2}}\right)=\frac{1}{\sqrt{x^2+y^2}}$$

$$z_{yy}=-\frac{1}{2}\frac{1}{\sqrt{(x^2+y^2)^3}}\cdot 2y=\frac{-y}{\sqrt{(x^2+y^2)^3}}$$

$$z_{xy}=\frac{-x}{\sqrt{(x^2+y^2)^3}}$$

习题8～4：P339.6（1）

$$z = f(2x, \frac{x}{y}) \qquad z_x = f_1' \cdot 2 + f_2' \cdot \frac{1}{y}$$

$$z_{xx} = 2[f_{11}'' \cdot 2 + f_{12}'' \cdot \frac{1}{y}] + \frac{1}{y} \cdot [f_{21}'' \cdot 2 + f_{22}'' \cdot \frac{1}{y}]$$

$$= 4f_{11}'' + \frac{4}{y} \cdot f_{12}'' \cdot 2 + \frac{1}{y^2} \cdot f_{22}''$$

习题8～4：P339．7

$$z = f(x + a\cdot t) + g(x - a\cdot t)$$

$$u = x + a\cdot t \quad v = x - a\cdot t$$

$$z_t = f_u \cdot a + g_v \cdot (-a) = a\cdot(f_u - g_v)$$

$$z_{tt} = a\cdot[f_{uu}\cdot a - g_{vv}\cdot(-a)] = a^2\cdot(f_{uu} + g_{vv})$$

$$z_x = f_u + g_v$$

$$z_{xx} = f_{uu} + g_{vv}$$

$$z_{tt} = a^2 \cdot z_{xx}$$ *ok!*

习题 8～5：P328. 1

$$x \cdot y - \ln y = e$$

$$y dx + x dy - \frac{1}{y} dy = 0$$

$$\frac{dy}{dx} = \frac{y}{\frac{1}{y} - x} = \frac{y^2}{1 - x \cdot y}$$

习题8～5：P328. 2

$$\ln\sqrt{x^2+y^2}=\arctan\frac{y}{x}$$

$$\frac{\dfrac{1}{2\sqrt{x^2+y^2}}\cdot(xdx+ydy)}{\sqrt{x^2+y^2}}=\frac{\dfrac{xdy-ydx}{x^2}}{1+\left(\dfrac{y}{x}\right)^2}$$

$$\frac{xdx+ydy}{x^2+y^2}=\frac{xdy-ydx}{x^2+y^2}\qquad \frac{dy}{dx}=\frac{x+y}{x-y}$$

习题８～５：Ｐ３２８．３

$$\sin(xy)+\cos(xz)+\tan(yz)=0$$

$$F_x=\cos(xy)\cdot y-\sin(xz)\cdot z$$

$$F_y=\cos(xy)\cdot x+\sec^2(xz)\cdot z$$

$$F_z=-\sin(xy)\cdot x+\sec^2(xz)\cdot y$$

$$\frac{\partial z}{\partial x}=-\frac{F_x}{Fz_x}=\frac{y\cdot\cos(xy)-z\cdot\sin(xz)}{x\cdot\sin(xy)-y\cdot\sec^2(xz)}$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{Fz_x}=\frac{x\cdot\cos(xy)+z\cdot\sec^2(xz)}{x\cdot\sin(xy)-y\cdot\sec^2(xz)}$$

JINSW 本科

习题8～5：P328．4

$$x+z=y\cdot f(x^2-z^2)$$

$$1+z_x=y\cdot f'\cdot(2x-2z\cdot z_x) \quad z_x=\frac{2xy\cdot f'-1}{1+2yz\cdot f'}$$

$$z_y=f+y\cdot f'\cdot(-2z\cdot z_y) \quad z_y=\frac{f}{1+2yz\cdot f'}$$

$$z\cdot z_x+y\cdot z_y$$

$$=\frac{1}{1+2yz\cdot f'}[z\cdot(2xy\cdot f'-1)+y\cdot f]$$

习题 8～5：P 3 4 5． 5

$$F(x, y, z) = 0$$

$$x = x(y, z); y = y(x, z); z = z(x, y)$$

$$\frac{\partial x}{\partial y} \cdot \frac{\partial y}{\partial z} \cdot \frac{\partial z}{\partial x} = \left(-\frac{F_y}{F_x}\right) \cdot \left(-\frac{F_z}{F_y}\right) \cdot \left(-\frac{F_x}{F_z}\right) = -1$$

习题８～５：Ｐ３４５．６

$$\frac{x}{z}=\ln\frac{z}{y} \qquad x=z\cdot\ln z-z\cdot\ln y$$

$$1=z_x\cdot\ln z+z\cdot\frac{1}{z}\cdot z_x-z_x\cdot\ln y$$

$$z_x=\frac{1}{\ln\frac{z}{y}+1}=\frac{1}{\frac{x}{z}+1}=\frac{z}{x+z}$$

$$z_{xx}=\frac{1}{(x+z)^2}\{z_x\cdot(x+z)-z\cdot(1+z_x)\}=\frac{-z^2}{(x+z)^3}$$

习题8～5：P345．6续

$$\frac{x}{z}=\ln\frac{z}{y} \qquad x=z\cdot\ln z-z\cdot\ln y$$

$$0=z_y\cdot\ln z+z\cdot\frac{1}{z}\cdot z_y-z_x\cdot\ln y-z\cdot\frac{1}{y}$$

$$z_y=\frac{z^2}{y(x+z)}$$

$$z_{yy}=\frac{2z\cdot z_y y(x+z)-z^2[(1+z)+y\cdot z_y]}{y^2(x+z)^2}$$

$$=\frac{-x^2\cdot z^2}{y^2\cdot(x+z)^3}$$

习题8～5：P345．7

$$e^z = x\cdot y\cdot z \quad e^z\cdot z_x = y\cdot z + x\cdot y\cdot z_x$$

$$z_x = \frac{y\cdot z}{e^z - x\cdot y} = \frac{z}{x\cdot z - x} \quad z_y = \frac{z}{y\cdot z - y}$$

$$z_{xy} = \frac{1}{x^2(z-1)^2}\{z_y\cdot x(z-1) - z\cdot x\cdot z_y\}$$

$$= \frac{-z}{x\cdot y\cdot (z-1)^3}$$

习题8～5：P345.10

$$\begin{cases} u = x - 2y \\ v = x + ay \end{cases} \quad \frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial x} = \frac{\partial z}{\partial u} + \frac{\partial z}{\partial v}$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v}\right) = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial u}\right) + \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial v}\right)$$

$$= \left[\frac{\partial}{\partial u}\left(\frac{\partial z}{\partial u}\right) \cdot \frac{\partial u}{\partial x} + \frac{\partial}{\partial v}\left(\frac{\partial z}{\partial u}\right) \cdot \frac{\partial v}{\partial x}\right] +$$

$$+ \left[\frac{\partial}{\partial u}\left(\frac{\partial z}{\partial v}\right) \cdot \frac{\partial u}{\partial x} + \frac{\partial}{\partial v}\left(\frac{\partial z}{\partial v}\right) \cdot \frac{\partial v}{\partial x}\right]$$

习题8～5：P345.10（续）

$$= \left[\frac{\partial}{\partial u}\left(\frac{\partial z}{\partial u}\right)\cdot\frac{\partial u}{\partial x}+\frac{\partial}{\partial v}\left(\frac{\partial z}{\partial u}\right)\cdot\frac{\partial v}{\partial x}\right]+\left[\frac{\partial}{\partial u}\left(\frac{\partial z}{\partial v}\right)\cdot\frac{\partial u}{\partial x}+\frac{\partial}{\partial v}\left(\frac{\partial z}{\partial v}\right)\cdot\frac{\partial v}{\partial x}\right]$$

$$=\frac{\partial^2 z}{\partial u^2}+2\frac{\partial^2 z}{\partial u\partial v}+\frac{\partial^2 z}{\partial v^2}$$

同理 $\dfrac{\partial^2 z}{\partial x\partial y}=-2\dfrac{\partial^2 z}{\partial u^2}+(-2+a)\dfrac{\partial^2 z}{\partial u\partial v}+a\dfrac{\partial^2 z}{\partial v^2}$

$$\frac{\partial^2 z}{\partial y^2}=4\frac{\partial^2 z}{\partial u^2}-4a\frac{\partial^2 z}{\partial u\partial v}+a^2\frac{\partial^2 z}{\partial v^2}$$

由条件得 $\begin{cases}6+a-a^2=0\\12+(-2+a)+4a\neq 0\end{cases}\Rightarrow a=3$

习题8～6：P337．1

$$f(x,y)=4(x-y)-x^2-y^2$$

$$\begin{cases} f_x=4-2x=0 \\ f_y=-4-2y=0 \end{cases} \qquad \begin{cases} x=2 \\ y=-2 \end{cases}$$

$$A=f_{xx}=-2, B=f_{xy}=0, C=f_{yy}=-2$$

$$B^2-AC=0-(-2)(-2)=-4<0$$

$$A\Big|_{(2,-2)}=-2<0$$

$$\max f(2,-2)=8$$

习题8～6：P354．3

$$z = x^2 - y^2 \qquad D: x^2 + 4y^2 \le 4$$

$$\begin{cases} z_x = 2x = 0 \\ z_y = -2y = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases}$$

$$A = z_{xx} = 2 \quad B = z_{xy} = 0 \quad C = z_{yy} = -2$$

$B^2 - AC = 4 > 0 \qquad (0,0)$ 不是极值点

极值必在D得边界上取得

$$F(x, y, \lambda) = x^2 - y^2 + \lambda(x^2 + 4y^2 - 4)$$

习题8～6：P354．3（续）

$$z = x^2 - y^2 \qquad D: x^2 + 4y^2 \le 4$$

$$F(x, y, \lambda) = x^2 - y^2 + \lambda(x^2 + 4y^2 - 4)$$

$$\begin{cases} F_x = 2(\lambda + 1)x = 0 \\ F_y = 2(4\lambda - 1)y = 0 \\ F_\lambda = x^2 + 4y^2 - 4 = 0 \end{cases}$$

$x = 0, y = 1$

or

$x = 2, y = 0$

$$z_{max} = z(2,0) = 4$$

$$z_{min} = z(0,1) = -1$$

习题8～6：P338．6

$$L(x,y)=R_1+R_2-C$$

$$=(18-2Q_1)Q_1+(12-Q_2)Q_2-[2(Q_1+Q_2)+5]$$

$$=-2Q_1^2+16Q_1-Q_2^2+10Q_2-5$$

$$\begin{cases} L_{Q_1}=-4Q_1+16=0 \\ L_{Q_2}=-2Q_2+10=0 \end{cases} \quad \begin{cases} Q_1=4 \\ Q_2=5 \end{cases} \quad \begin{cases} P_1=10 \\ P_2=7 \end{cases}$$

$$L_{Q_1Q_1}=-4<0 \quad L_{Q_1Q_2}=0 \quad L_{Q_2Q_2}=-2$$

$$B^2-AC=-8<0$$

$$\max \quad L(4,5)=52$$

JINSW 本科

习题8～6：P338．6（续）

$$P_1 = P_2 \quad Q_2 = 2Q_1 - 6$$

$$L = -6Q_1^2 + 60Q_1 - 101$$

$$L' = -12Q_1 + 60 = 0 \quad Q_1 = 5$$

$$L'' = -12 < 0$$

$$Q_2 = 4 \quad P_1 = P_2 = 8$$

$$\max \quad L(5,4) = 49$$

习题８～６：Ｐ３３８．８

$$R = 15 + 14x_1 + 32x_2 - 8x_1x_2 - 2x_1^2 - 10x_2^2$$

$$\begin{cases} R_{x_1} = 14 - 8x_2 - 4x_1 = 0 \\ R_{x_1} = 32 - 8x_1 - 20x_2 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 1.5 \\ x_2 = 1 \end{cases}$$

$$R_{x_1x_1} = -4 < 0 \quad R_{x_1x_2} = -8 \quad R_{x_2x_2} = -20$$

$$B^2 - AC = -16 < 0 \qquad \max$$

$$x_2 = 1.5 - x_1$$

$$R = 15 + 14x_1 + 32(1.5 - x_1) - 8x_1(1.5 - x_1) - 2x_1^2 - 10(1.5 - x_1)^2$$

$$R = 40.5 - 4x_1^2 \quad R' = -8x_1 = 0 \Rightarrow x_1 = 0$$

$$R'' = -8 < 0 \quad \max$$

习题8～6：P355．10

点到直线的距离 $d=\dfrac{|Ax+By+C|}{\sqrt{A^2+B^2}}=\dfrac{|x-y-2|}{\sqrt{1^2+(-1)^2}}$

点在抛物线上 $y=x^2$

目标函数 $F(x,y,\lambda)=\dfrac{1}{\sqrt{2}}|x-y-2|+\lambda(y-x^2)$

等价 $F(x,y,\mu)=x-y-2+\mu(y-x^2)$

$$\begin{cases} F_x=1-2\mu x=0 \\ F_y=-1+\mu=0 \\ F_\mu=y-x^2=0 \end{cases} \quad \begin{cases} x=0.5 \\ \mu=1 \\ y=0.25 \end{cases} \quad d_{\min}=\dfrac{|0.5-.025-2|}{\sqrt{2}}=\dfrac{7}{8}\sqrt{2}$$

总习题八：Ｐ３６１．７

$$u = x\cdot y\cdot z\cdot e^{x+y+z} \qquad \frac{\partial^{p+q+r}u}{\partial^p x\partial^q y\partial^r z} \qquad (p,q,r\in N)$$

$$u=(xe^x)\cdot(ye^y)\cdot(ze^z)$$

$$\frac{\partial^p u}{\partial x^p}=(ye^y)\cdot(ze^z)\cdot\sum_{k=0}^{p}C_p^k\cdot(x)^{(k)}\cdot(e^x)^{(p-k)}$$

$$=(ye^y)\cdot(ze^z)\cdot(x+p)e^x$$

$$\frac{\partial^{p+q}u}{\partial x^p\partial y^q}=\frac{\partial^q}{\partial y^q}\left(\frac{\partial^p u}{\partial x^p}\right)$$

$$=\frac{\partial^q}{\partial y^q}\left((ye^y)\cdot(ze^z)\cdot(x+p)e^x\right)$$

总习题八：Ｐ３６１．７（续）

$$\frac{\partial^{p+q}u}{\partial x^p \partial y^q} = \frac{\partial^q}{\partial y^q}\Big((ye^y)\cdot(ze^z)\cdot(x+p)e^x\Big)$$

$$= (ze^z)\cdot(x+p)e^x \cdot \frac{\partial^q (ye^y)}{\partial y^q}$$

$$= (ze^z)\cdot(x+p)e^x \cdot (y+q)e^y$$

$$\frac{\partial^{p+q+r}u}{\partial x^p \partial y^q \partial z^r} = \frac{\partial^r}{\partial z^r}\Big((y+q)e^y \cdot (ze^z)\cdot(x+p)e^x\Big)$$

$$= (y+q)e^y \cdot (z+r)e^z \cdot (x+p)e^x$$

$$= (x+p)\cdot(y+q)\cdot(z+r)e^{x+y+z}$$

总习题八：Ｐ３６１．８

$$\begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \end{cases} \qquad z = f(x, y)$$

$$\begin{cases} f_\rho = f_x\cos\theta + f_y\sin\theta \\ f_\theta = f_x(-\rho\sin\theta) + f_y\rho\cos\theta \end{cases}$$

$$xf_x + yf_y = 0 \Rightarrow \rho \cdot f_\rho = 0 \Rightarrow f_\rho = 0$$

$$f(x, y) = f(\theta)$$

总习题八：P361. 9

$$F(x,y)=f(x)+g(y)=S(\rho)\quad \begin{cases} x=\rho\cos\theta \\ y=\rho\sin\theta \end{cases}$$

$$\frac{\partial F}{\partial \theta}=0$$

$$\Rightarrow f'(x)\cdot\rho(-\sin\theta)+g'(y)\cdot\rho\cos\theta=0$$

$$\Rightarrow -y\cdot f'(x)+x\cdot g'(y)=0$$

$$\Rightarrow y\cdot f'(x)=x\cdot g'(y)$$

$$\Rightarrow f'(x)=kx, g'(y)=ky \quad (k\in R)$$ （关键）

$$f(x)=0.5kx^2+C_1 \quad g(y)=0.5ky^2+C_2$$

$$F(x,y)=C(x^2+y^2)+C_0 \quad (C,C_0\in R)$$

总习题八：P362. 10　　2－1

$$f(x,y)=\begin{cases}\dfrac{x^2y^2}{(x^2+y^2)^{\frac{3}{2}}} & x^2+y^2\neq 0\\ 0 & x^2+y^2=0\end{cases}$$

$$|f(x,y)-f(0,0)|=\frac{x^2y^2}{(x^2+y^2)^{\frac{3}{2}}}$$

$$\leq\frac{(x^2+y^2)(x^2+y^2)}{(x^2+y^2)^{\frac{3}{2}}}=\sqrt{x^2+y^2}\to 0$$ 连续

总习题八：P362．10　　2－2

$$f_x(0,0)=\lim_{\Delta x\to 0}\frac{f(\Delta x,0)-f(0,0)}{\Delta x}=0$$

可导

$$f_y(0,0)=\lim_{\Delta y\to 0}\frac{f(0,\Delta y)-f(0,0)}{\Delta y}=0$$

$$\frac{|\Delta z-dz|}{\rho}=\frac{(\Delta x)^2\cdot(\Delta y)^2}{[(\Delta x)^2+(\Delta y)^2]^{\frac{3}{2}}\sqrt{(\Delta x)^2+(\Delta y)^2}}$$

$$=\frac{(\Delta x)^2\cdot(\Delta y)^2}{[(\Delta x)^2+(\Delta y)^2]^2}\xrightarrow{\rho\to 0}\not\exists$$

不可微

总习题八：P362．11　　（3－1）本科

$$z = f(2x-y) + g(x, xy) \quad u = 2x - y, v = xy$$

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}\left(f(u) + g(x,v)\right) = \frac{\partial}{\partial x}\left(f(u)\right) + \frac{\partial}{\partial x}\left(g(x,v)\right)$$

$$\frac{\partial}{\partial x}\left(f(u)\right) = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x} = f'_u \cdot 2$$

$$\frac{\partial}{\partial x}\left(g(x,v)\right) = \frac{\partial g(x,v)}{\partial x} + \frac{\partial g}{\partial v}\cdot\frac{\partial v}{\partial x} = g'_1 + g'_2 \cdot y$$

$$\frac{\partial z}{\partial x} = 2f'_u + g'_1 + y\cdot g'_2$$

总习题八：P362.11（续）（3—2）

$$\frac{\partial z}{\partial x}=2f'_u+g'_1+y\cdot g'_2$$

$$\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial}{\partial y}(2f'_u+g'_1+y\cdot g'_2)$$

$$=2\frac{\partial}{\partial y}(f'_u)+\frac{\partial}{\partial y}(g'_1)+\frac{\partial}{\partial y}(y\cdot g'_2)$$

$$\frac{\partial}{\partial y}(f'_u)=\frac{\partial f'}{\partial u}\cdot\frac{\partial u}{\partial y}=f''_{uu}\cdot(-1)$$

$$\frac{\partial}{\partial y}(g'_1)=\frac{\partial g'_1}{\partial v}\cdot\frac{\partial v}{\partial y}=g''_{12}\cdot x$$

总习题八：P362．11（续）（3—3）

$$\frac{\partial^2 z}{\partial x \partial y} = 2\frac{\partial}{\partial y}(f_u') + \frac{\partial}{\partial y}(g_1') + \frac{\partial}{\partial y}(y \cdot g_2')$$

$$\frac{\partial}{\partial y}(f_u') = f_{uu}'' \cdot (-1) \qquad \frac{\partial}{\partial y}(g_1') = g_{12}'' \cdot x$$

$$\frac{\partial}{\partial y}(y \cdot g_2') = g_2' + y \cdot \frac{\partial g_2'}{\partial v} \cdot \frac{\partial v}{\partial y} = g_2' + y \cdot g_{22}'' \cdot x$$

$$\frac{\partial^2 z}{\partial x \partial y} = -2 f_{uu}'' + x \cdot g_{12}'' + g_2' + x \cdot y \cdot g_{22}''$$

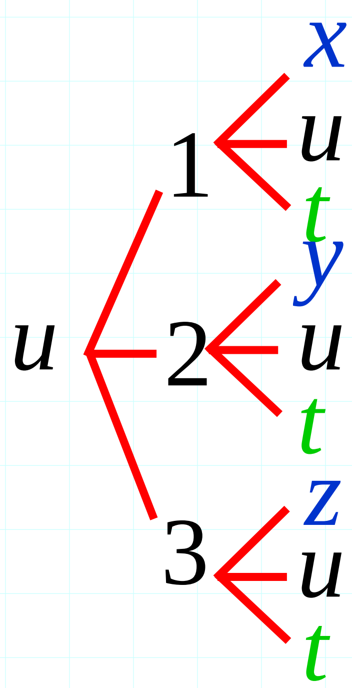
总习题八：Ｐ３６２．１２ （２－１）

$$\begin{cases} u = f(x-u\cdot t,\, y-u\cdot t,\, z-u\cdot t) \\ g(x,y,z)=0 \end{cases}$$

方程组确定二个三元隐函数 $\begin{cases} u = u(x,y,t) \\ z = z(x,y,t) \end{cases}$

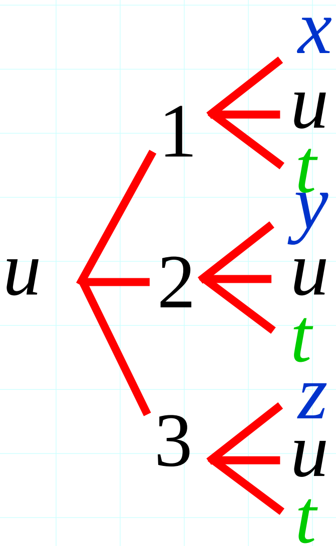
变量关系图

$$u = f(x-u\cdot t,\, y-u\cdot t,\, z-u\cdot t)$$



总习题八：P362. 12 （2－2）

$$\begin{cases} u = f(x-u\cdot t,\, y-u\cdot t,\, z-u\cdot t) \\ g(x,y,z)=0 \end{cases}$$



$$\begin{cases} u_x = f_1\cdot(1-t\cdot u_x)+ \\ \qquad + f_2\cdot(-t\cdot u_x)+ \\ \qquad + f_3\cdot(z_x - t\cdot u_x) \\ z_x = -\dfrac{g_x}{g_z} \end{cases}$$

$$u_x = \frac{g_z\cdot f_1 - g_x\cdot f_3}{g_z\cdot[t\cdot(f_1+f_2+f_3)]} \qquad u_y = \frac{g_z\cdot f_2 - g_x\cdot f_3}{g_z\cdot[t\cdot(f_1+f_2+f_3)]}$$

总习题八：P362. 13

余弦定理 $\cos A = \dfrac{b^2+c^2-a^2}{2bc}$

两边关于 a 求导

$$-\sin A \cdot \frac{\partial A}{\partial a} = \frac{-2a}{2bc} = \frac{-a}{bc} \Rightarrow \frac{\partial A}{\partial a} = \frac{a}{b\sin A}$$

两边关于 b 求导

$$-\sin A \cdot \frac{\partial A}{\partial b} = \frac{4b^2c - 2c(b^2+c^2-a^2)}{4b^2c^2} = \frac{b - c\cos A}{bc}$$

$$\Rightarrow \frac{\partial A}{\partial b} = \frac{c\cos A - b}{bc\sin A}$$ 同理 $$\frac{\partial A}{\partial c} = \frac{b\cos A - c}{bc\sin A}$$

总习题八：P362．15

$$\begin{cases} f_x = -(1+e^y)\cdot\sin x = 0 \\ f_y = e^y\cdot(\cos x - 1 - y) = 0 \end{cases}$$

$$\begin{cases} x_0 = k\pi \quad k\in Z \\ y_0 = \cos x_0 - 1 = \begin{cases} 0 & x_0 = 2n\pi \quad n\in Z \\ -2 & x_0 = (2n+1)\pi \quad n\in Z \end{cases} \end{cases}$$

$$A = f_{xx} = \begin{cases} -2 & x_0 = 2n\pi \quad n\in Z \\ 1+e^{-2} & x_0 = (2n+1)\pi \quad n\in Z \end{cases}$$

$$B = f_{xy} = 0$$

$$C = f_{yy} = \begin{cases} -1 & x_0 = 2n\pi \quad n\in Z \\ -5e^{-2} & x_0 = (2n+1)\pi \quad n\in Z \end{cases}$$

总习题八：P362．15（续）

$$A = f_{xx} = \begin{cases} -2 & x_0 = 2n\pi \quad n \in Z \\ 1+e^{-2} & x_0 = (2n+1)\pi \quad n \in Z \end{cases}$$

$$B = f_{xy} = 0$$

$$C = f_{yy} = \begin{cases} -1 & x_0 = 2n\pi \quad n \in Z \\ -5e^{-2} & x_0 = (2n+1)\pi \quad n \in Z \end{cases}$$

$x_0 = 2n\pi \quad n \in Z \quad B^2 - AC < 0, A < 0$ 极大

$x_0 = (2n+1)\pi \quad n \in Z \quad B^2 - AC > 0$ 非极值

函数有无穷多个极大值，无极小值。

总习题八：P 3 6 2 . 1 6　（2－1）

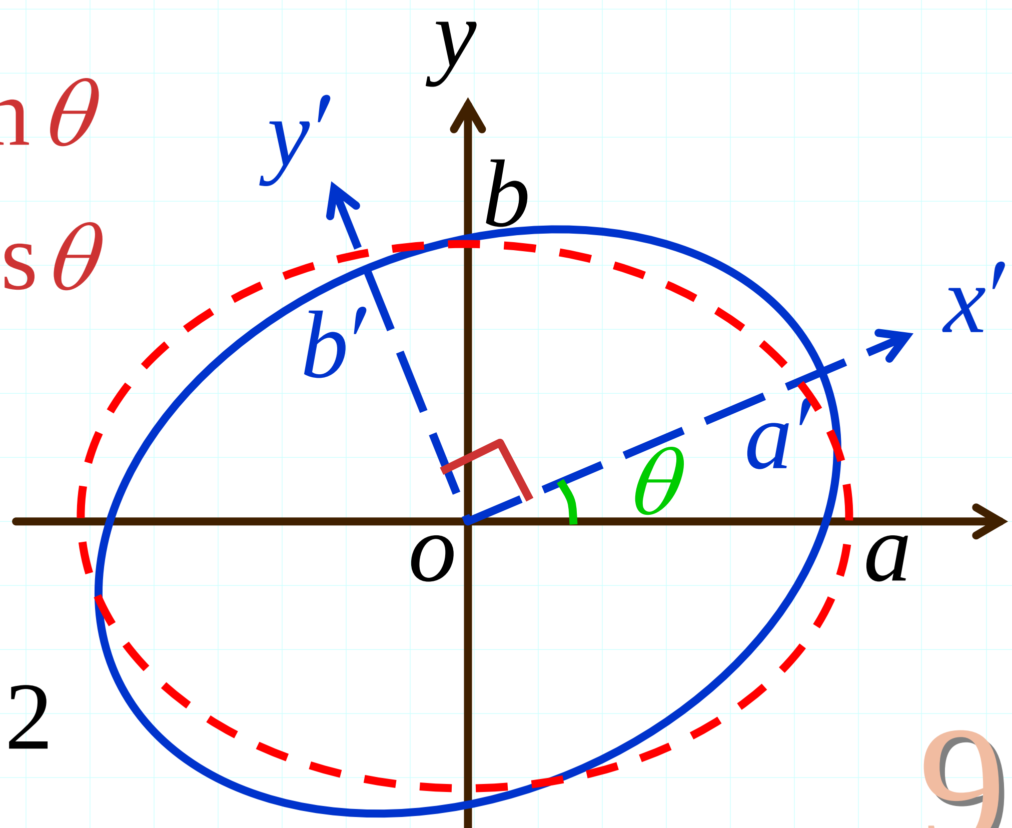
$$5x^2 + 4xy + 2y^2 = 1$$ 椭圆 $$Ax^2 + 2Bxy + Cy^2 = 1$$

标准椭圆 $$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad S = \pi \cdot a \cdot b$$

$$\begin{cases} x = x'\cos\theta - y'\sin\theta \\ y = x'\sin\theta + y'\cos\theta \end{cases}$$

$$\cot 2\theta = \frac{A - C}{2B}$$

$$A = 5, 2B = 4, C = 2$$



总习题八：P362. 16 (2－2)

$$5x^2+4xy+2y^2=1$$

$$\begin{cases} x = x'\cos\theta - y'\sin\theta \\ y = x'\sin\theta + y'\cos\theta \end{cases}$$

$$A=5, 2B=4, C=2$$

$$\cot 2\theta = 0.75$$

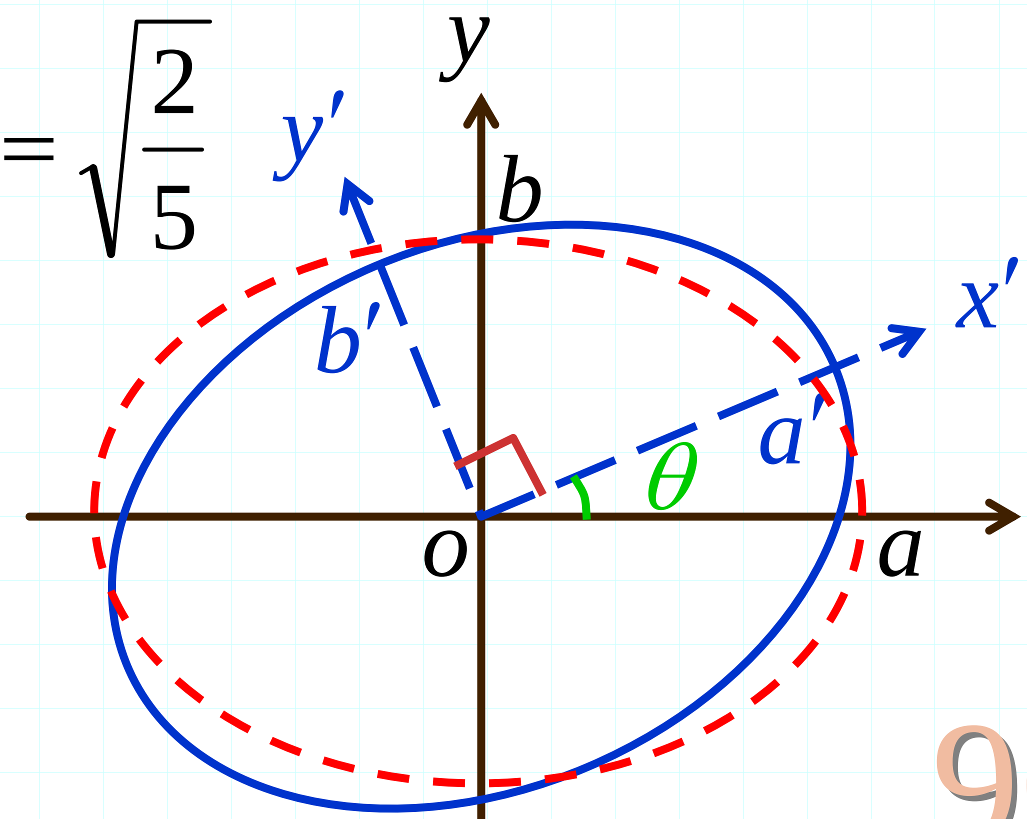
$$\sin\theta = \sqrt{\frac{1}{5}} \quad \cos\theta = \sqrt{\frac{2}{5}}$$

$$6(x')^2+(y')^2=1$$

$$a' = \frac{1}{\sqrt{6}}$$

$$b'=1$$

$$S = \frac{\pi}{\sqrt{6}}$$



总习题八：P362. 19

$$x + y = 230 \Rightarrow x = 230 - y$$

$$Q = -8x^2 + 12xy - 3y^2$$

$$= -8(230 - y)^2 + 12(230 - y)y - 3y^2$$

$$= -23y^2 + 6440y - 423200$$

$$Q'_y = -46y + 6440 = 0 \Rightarrow y = 140, x = 90$$

$$Q''_{yy} = -46 < 0$$

$$Q_{max} = Q(90,140) = 27600$$

习题9～1：P351．1

$$I_1 = \iint_{D_1} (x^2 + y^2)^3 d\sigma$$

$$D_1 = \{(x, y) \mid |x| \le 1, |y| \le 2\}$$

$$I_2 = \iint_{D_2} (x^2 + y^2)^3 d\sigma$$

$$D_2 = \{(x, y) \mid 0 \le x \le 1, 0 \le y \le 2\}$$

$z = f(x, y)$ 满足

$$z = f(-x, -y) = f(x, y) = f(-x, y) = f(x, -y)$$

且 $D_1 = 4D_2$ 则 $I_1 = 4I_2$

习题 9～1：P351．3（1）

$$I_1=\iint_D (x+y)^2 d\sigma \qquad I_2=\iint_D (x+y)^3 d\sigma$$

$$D=\{(x,y)\,|\,x\ge 0, y\ge 0, x+y\le 1\}$$

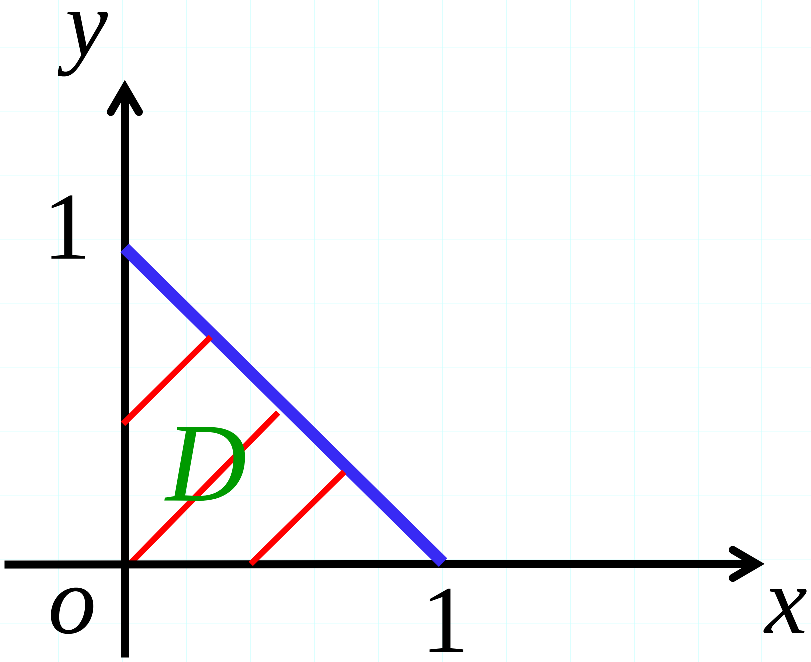
在D上处处有

$$0\le (x+y)\le 1$$

在D上处处有

$$(x+y)^2\ge (x+y)^3$$

$$I_1\ge I_2$$



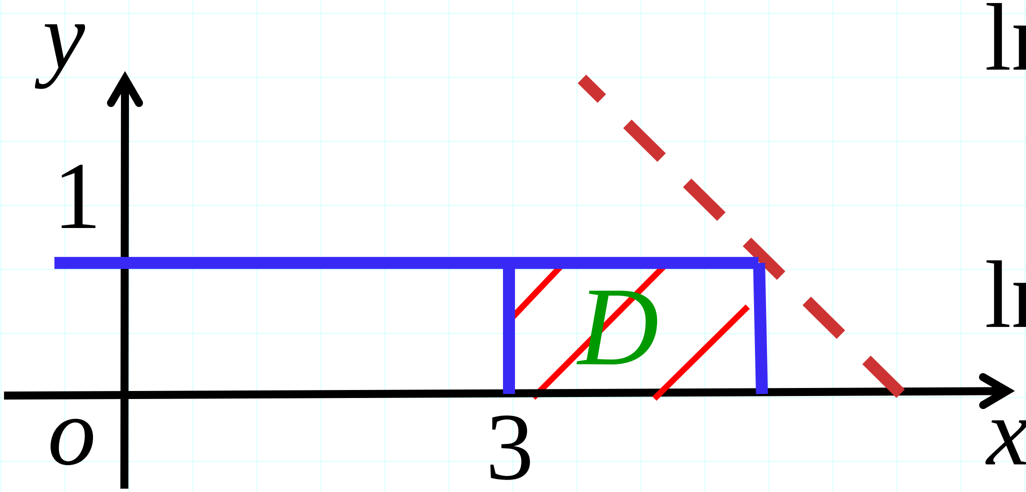
习题 9～1：P351．3（2）

$$I_1 = \iint_D \ln(x+y)d\sigma \qquad I_2 = \iint_D \ln^2(x+y)d\sigma$$

$$D = \{(x,y) \mid 3 \le x \le 5, 0 \le y \le 1\}$$

$$\underset{D}{Max}(x+y) = (x+y)\Big|_{(5,1)} = 6$$

$$\underset{D}{Min}(x+y) = (x+y)\Big|_{(3,0)} = 3$$



$$\ln 3 \le \ln(x+y) \le \ln 6$$

$$\ln(x+y) > 1$$

$$\ln(x+y) < \ln^2(x+y)$$

$$I_1 < I_2$$

习题9～1：P351.4（1）

$$I=\iint\limits_{D} x\cdot y\cdot(x+y+1)d\sigma$$

$$D=\{(x,y)\,|\,0\le x\le 1, 0\le y\le 2\}$$

$$\underset{D}{Max}\,z=z\big|_{(1,2)}=1\times 2\times(1+2+1)=8$$

$$\underset{D}{Min}\,z=z\big|_{(0,0)}=0$$

$$A=2$$

$$0\le I\le 16$$

习题９～１：Ｐ３５２．４（２）

$$I=\iint\limits_{D}(x^2+4y^2+9)d\sigma$$

$$D=\{(x,y)\,|\,x^2+y^2\le 4\}$$

$$9\le x^2+4y^2+9\le 4(x^2+y^2)+9\le 25$$

$$A=4\pi$$

$$36\pi\le I\le 100\pi$$

习题9～1：P352．4（3）

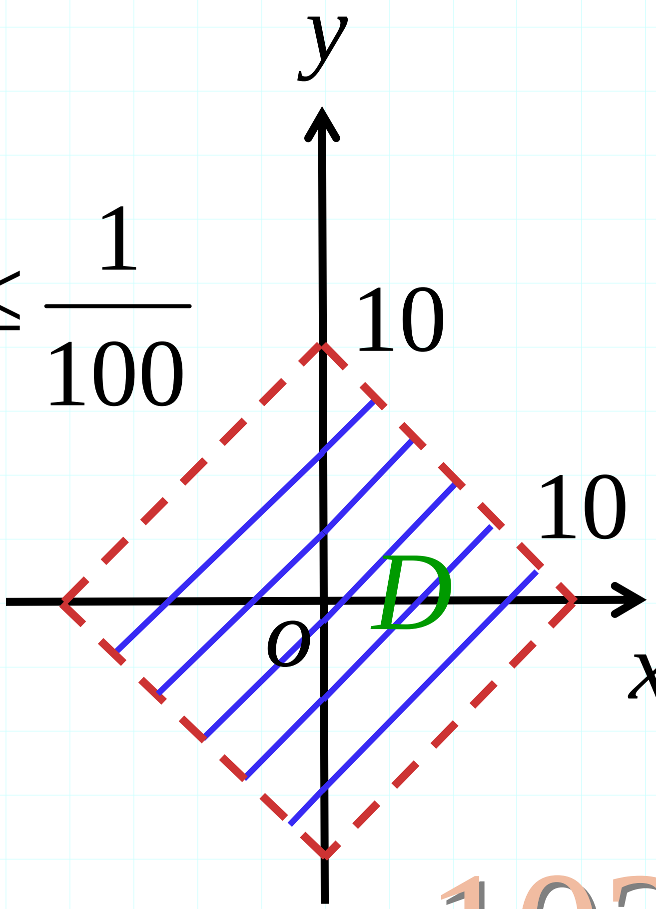
$$I=\iint\limits_{D}\frac{d\sigma}{100+\cos^2 x+\cos^2 y}$$

$$D=\{(x,y)\,|\,|x|+|y|\le 10\}$$

$$\frac{1}{102}\le\frac{1}{100+\cos^2 x+\cos^2 y}\le\frac{1}{100}$$

$$A=(10\sqrt{2})^2=200$$

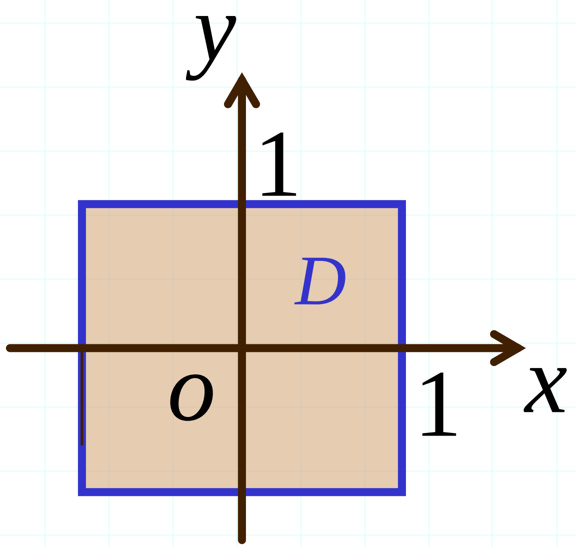
$$\frac{100}{51}\le I\le 2$$



习题9～2：P365．1（1）

$$\iint_D (x^2+y^2)d\sigma \quad D=\{(x,y)\,|\,|x|\le 1, |y|\le 1\}$$

$$D:\begin{cases}-1\le x\le 1\\-1\le y\le 1\end{cases}$$



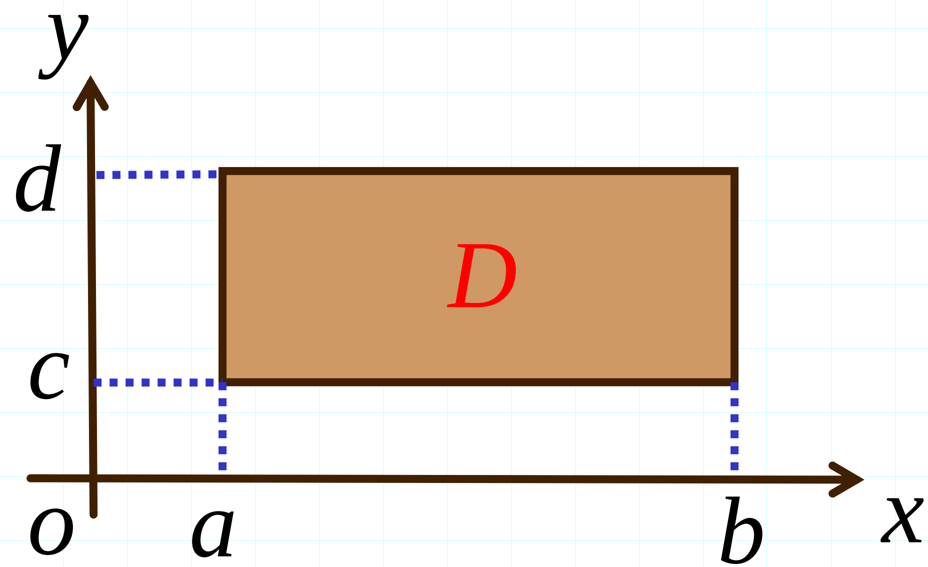
$$\text{原式}=\int_{-1}^{1}dx\int_{-1}^{1}(x^2+y^2)dy$$

$$=\int_{-1}^{1}\left[x^2y+\frac{1}{3}y^3\right]_{-1}^{1}dx=\left[\frac{2}{3}x^3+\frac{2}{3}x\right]_{-1}^{1}=\frac{8}{3}$$

习题9～2：P365．1（2）

$$\iint_D xye^{x^2+y^2}\,d\sigma$$

$$D:\begin{cases}0\le x\le a\\ 0\le y\le b\end{cases}$$



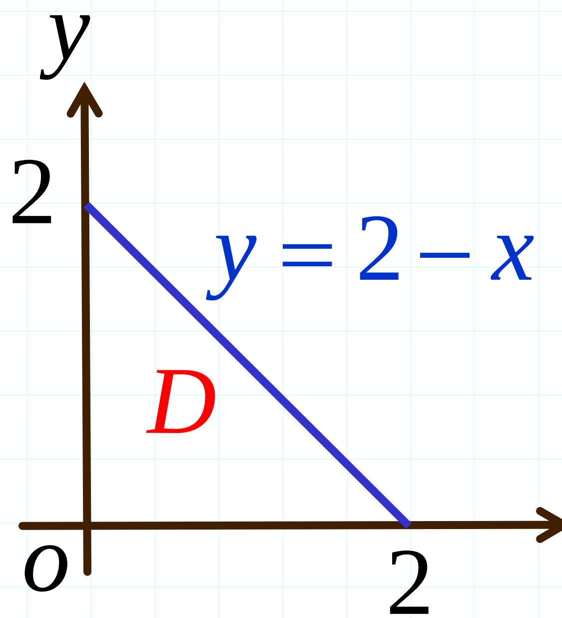
$$\text{原式}=\int_a^b xe^{x^2}dx\int_c^d ye^{y^2}dy$$

$$=\frac{1}{2}\int_a^b e^{x^2}d(x^2)\cdot\frac{1}{2}\int_c^d e^{y^2}d(y^2)$$

$$=\frac{1}{4}\cdot(e^{b^2}-e^{a^2})\cdot(e^{d^2}-e^{c^2})$$

习题9～2：P365.1（3）

$$\iint_D (3x+2y)d\sigma$$



$$D:\begin{cases} 0\le x\le 2 \\ 0\le y\le 2-x \end{cases}$$

$$\text{原式}=\int_0^2 dx\int_0^{2-x}(3x+2y)dy$$

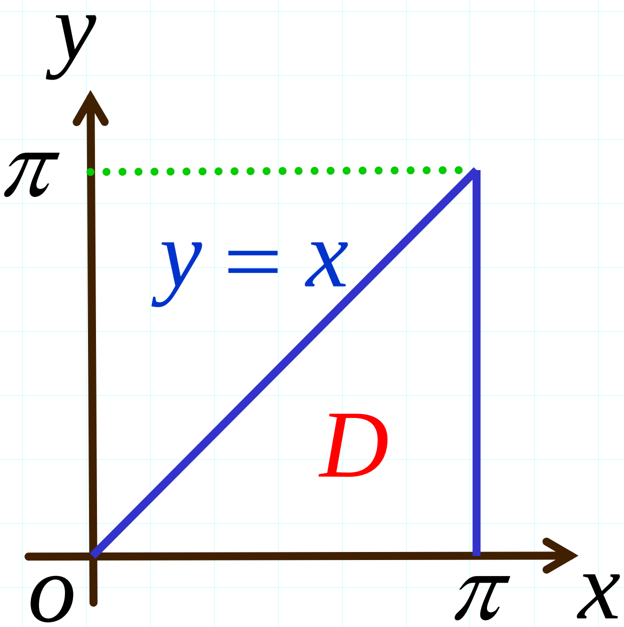
$$=\int_0^2 (3xy+y^2)\Big|_0^{2-x}dx$$

$$=\int_0^2 (4+2x-2x^2)dx=\frac{20}{3}$$

习题9～2：P365．1（4）

$$\iint_D x\cdot\cos(x+y)d\sigma$$

$$D:\begin{cases}0\le x\le\pi\\0\le y\le x\end{cases}$$



$$\text{原式}=\int_0^\pi dx\int_0^x x\cdot\cos(x+y)dy$$

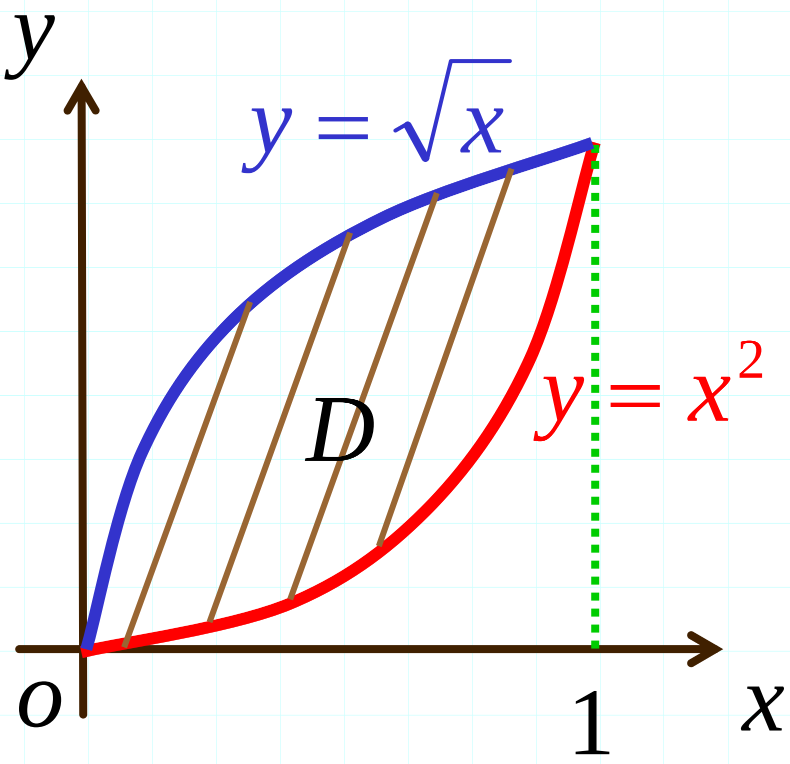
$$=\int_0^\pi x\cdot\sin(x+y)\big|_0^x\, dx$$

$$=\int_0^\pi (x\cdot\sin 2x-x\cdot\sin x)dx=-\frac{3}{2}\pi$$

习题9～2：P365．2（1）

$$\iint_D x\sqrt{y}d\sigma$$

$$D:\left\{\begin{matrix} 0\le x\le 1 \\ x^2\le y\le \sqrt{x} \end{matrix}\right\}$$



$$\text{原式}=\int_0^1 dx\int_{x^2}^{\sqrt{x}} x\cdot\sqrt{y}dy$$

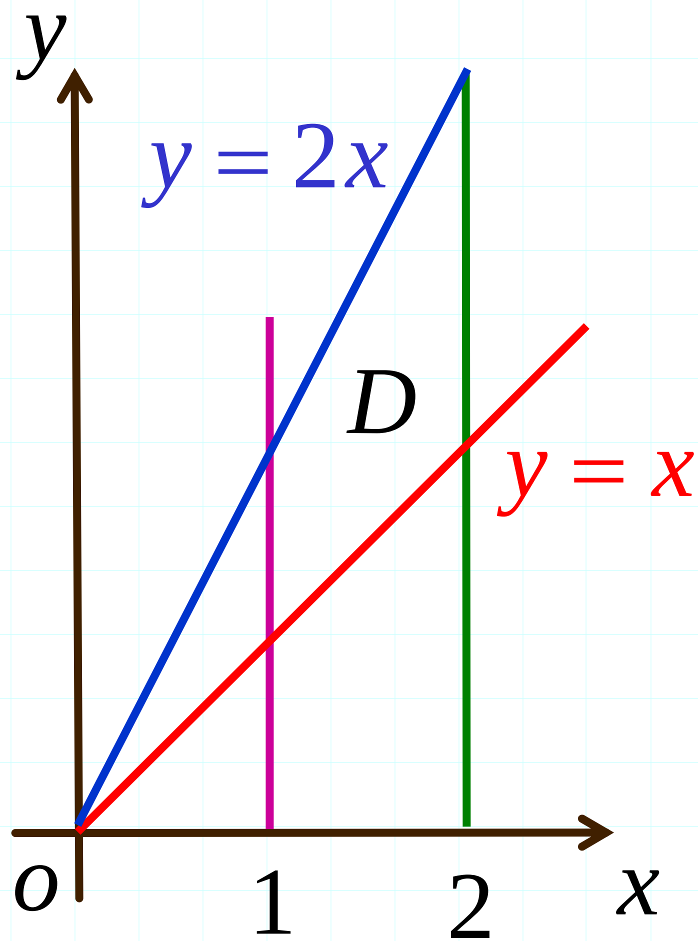
$$=\int_0^1 \frac{2}{3}x\cdot y^{\frac{3}{2}}\Big|_{x^2}^{\sqrt{x}}\ dx$$

$$=\frac{2}{3}\left(\frac{4}{11}x^{\frac{11}{4}}-\frac{1}{5}x^5\right)\Big|_0^1=\frac{6}{55}$$

习题9～2：P365. 2（2）

$$\iint_D \frac{y}{x} d\sigma$$

$$D:\begin{Bmatrix} 1 \le x \le 2 \\ x \le y \le 2x \end{Bmatrix}$$



$$\text{原式} = \int_1^2 dx \int_x^{2x} \frac{y}{x} dy$$

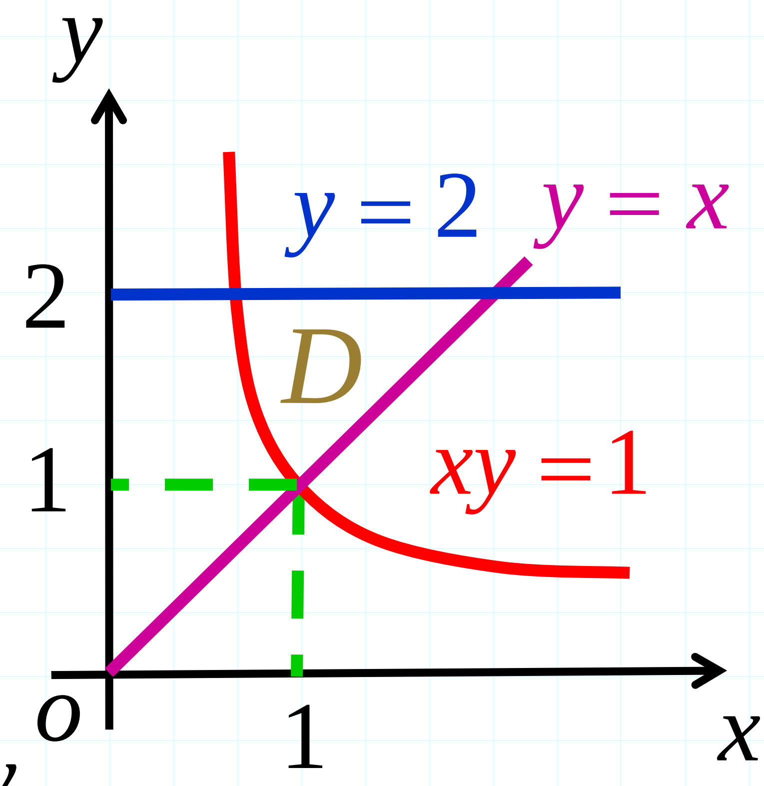
$$= \int_1^2 \frac{3}{2} x dx$$

$$= \frac{3}{4} x^2 \Big|_1^2 = \frac{9}{4}$$

习题9～2：P365．2（3）

$$\iint_D (2x+y)d\sigma$$

$$D:\left\{\begin{array}{l} 1\le y\le 2 \\ \dfrac{1}{y}\le x\le y \end{array}\right\}$$



$$\text{原式}=\int_1^2 dy\int_{\frac{1}{y}}^{y}(2x+y)dx$$

$$=\int_1^2 (2y^2-\frac{1}{y^2}-1)dy$$

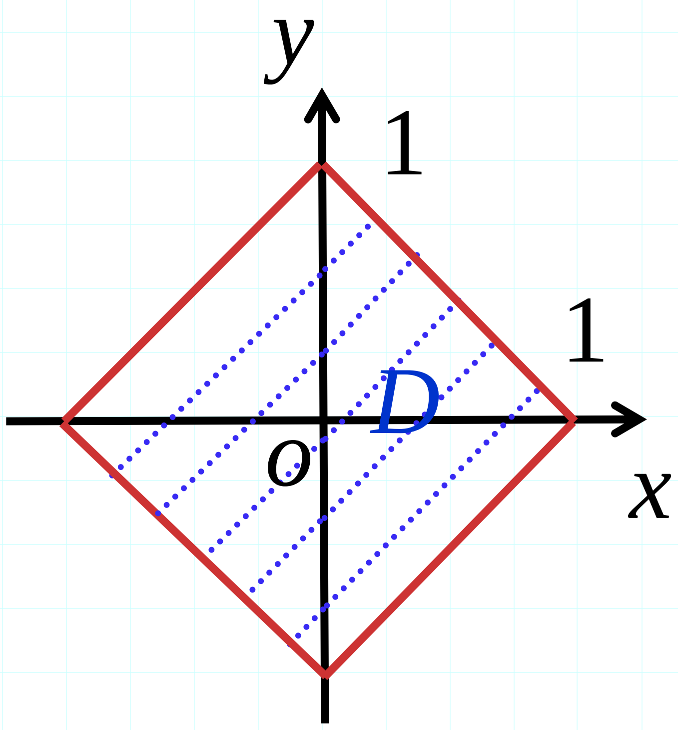
$$=(\frac{2}{3}y^3+\frac{1}{y}-y)\Big|_1^2=\frac{19}{6}$$

习题9～2：P365．2（4）

$$\iint_D e^{x+y}d\sigma$$

$$D:\left\{\begin{array}{c}-1\le x\le 0\\ -x-1\le y\le x+1\end{array}\right\}\cup$$

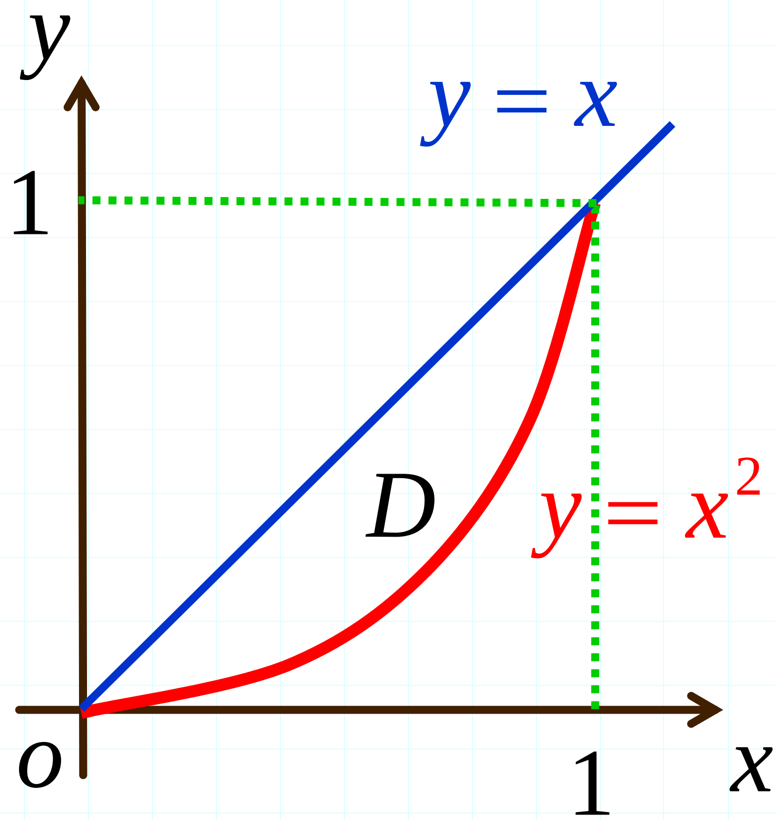
$$\cup\left\{\begin{array}{c}0\le x\le 1\\ x-1\le y\le -x+1\end{array}\right\}$$



$$原式=\int_{-1}^{0}dx\int_{-x-1}^{x+1}e^{x+y}dy+\int_{0}^{1}dx\int_{x-1}^{-x+1}e^{x+y}dy$$

$$=e-\frac{1}{e}$$

习题9～2：P365．5（1）



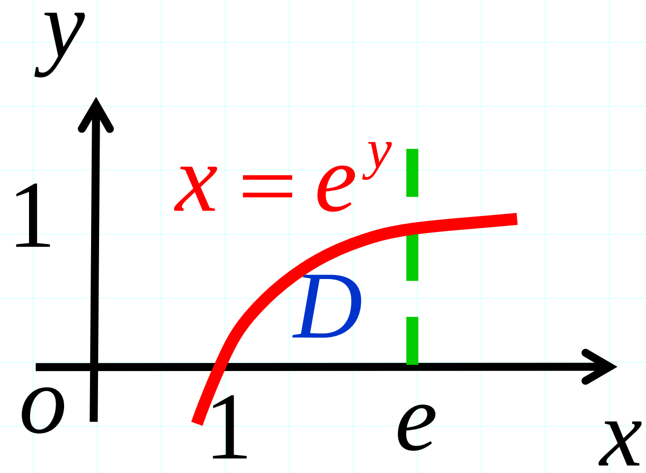
$$\int_0^1 dy\int_y^{\sqrt{y}} f(x,y)dx$$

$$=\int_0^1 dx\int_{x^2}^{x} f(x,y)dy$$

习题9～2：P365．5（2）

$$\int_0^1 dy \int_{e^y}^{e} f(x,y)\,dx$$

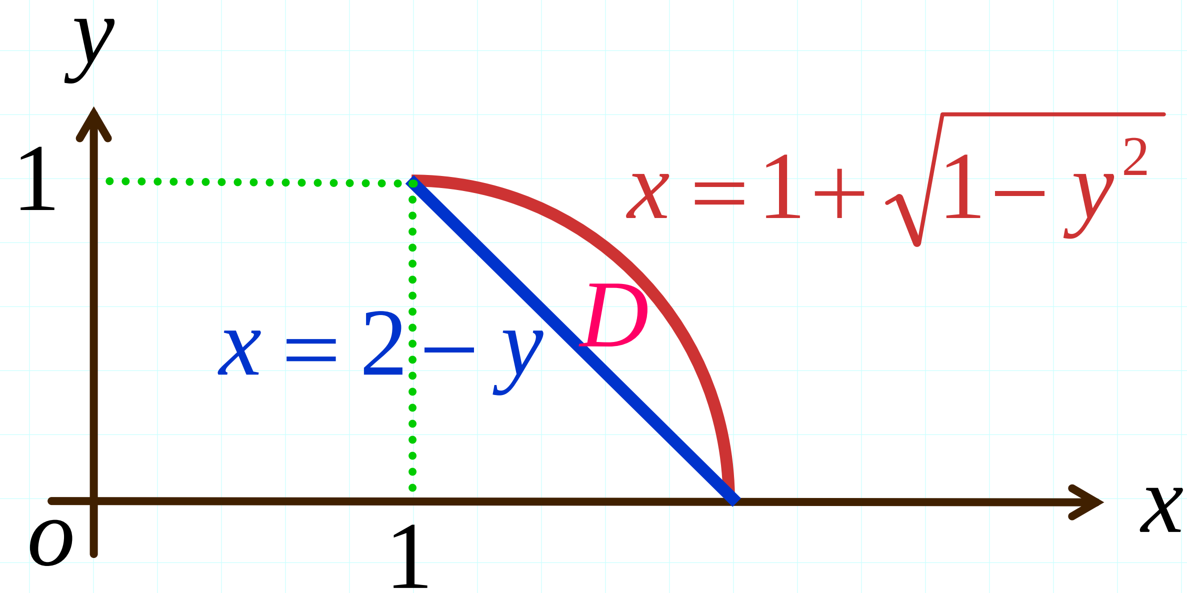
$$= \int_1^e dx \int_0^{\ln x} f(x,y)\,dy$$



习题9～2：P366. 5（3）

$$\int_0^1 dy \int_{2-y}^{1+\sqrt{1-y^2}} f(x,y)\,dx$$

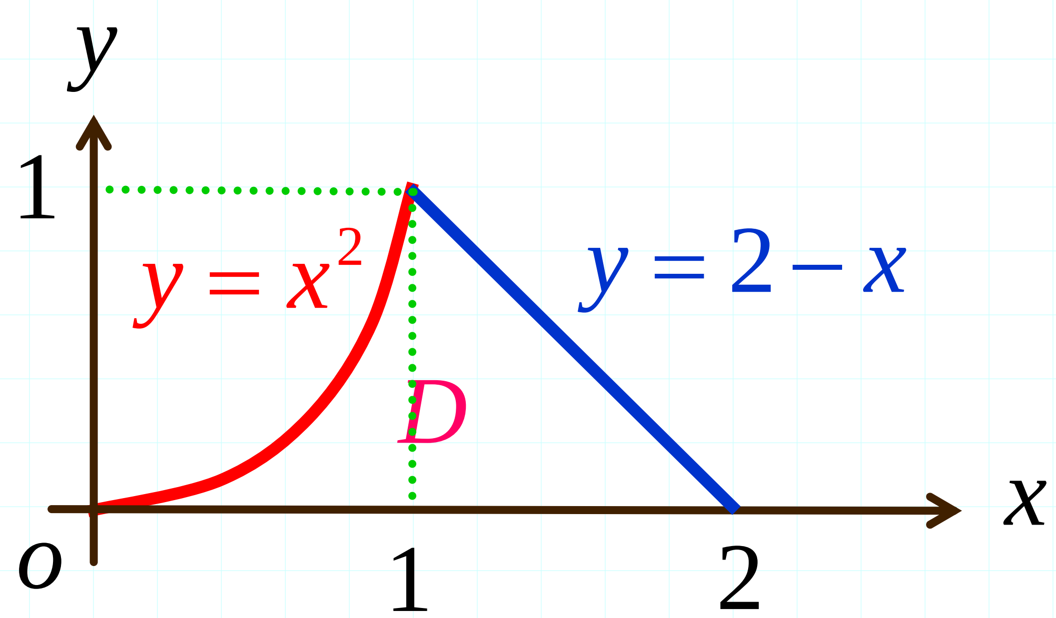
$$= \int_1^2 dx \int_{2-x}^{\sqrt{1-(x-1)^2}} f(x,y)\,dy$$



习题9～2：P366.5（4）

$$\int_0^1 dx \int_0^{x^2} f(x,y)dy + \int_1^2 dx \int_0^{2-x} f(x,y)dy$$

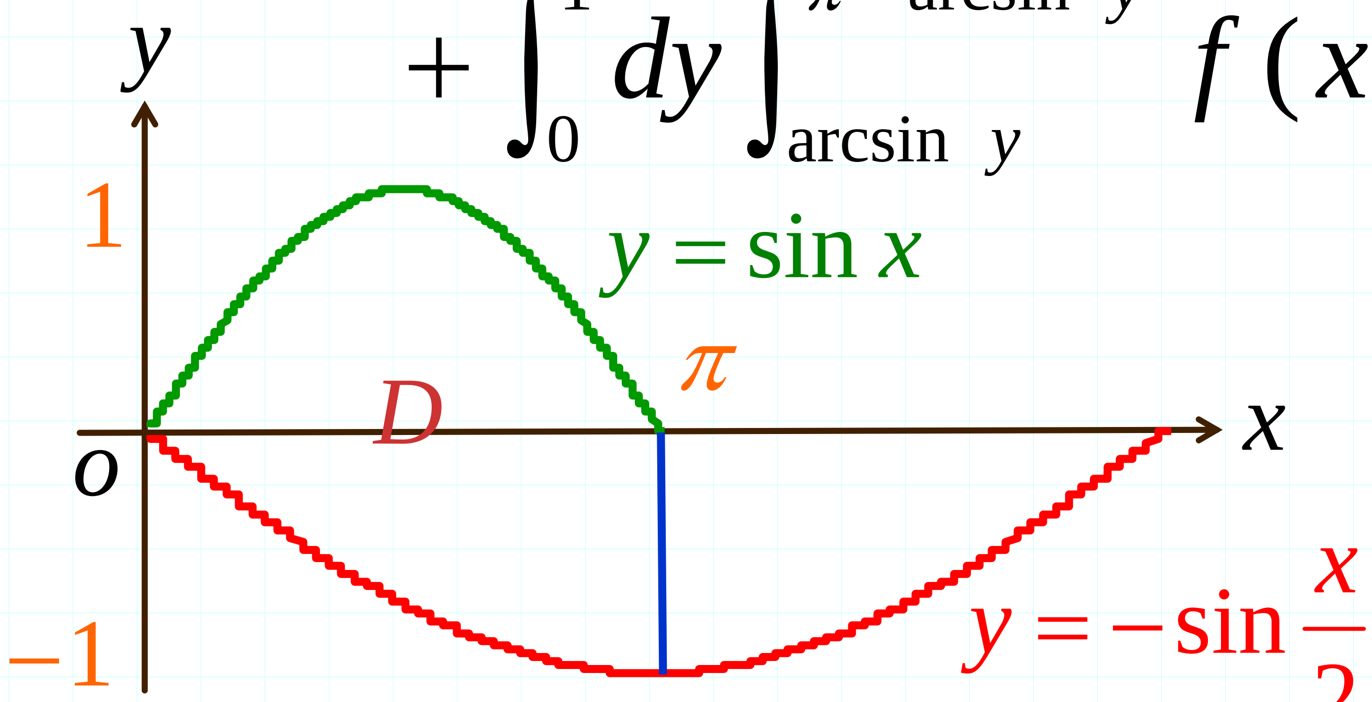
$$= \int_0^1 dy \int_{\sqrt{y}}^{2-y} f(x,y)dx$$



习题9～2：P366．5（5）

$$\int_0^{\pi} dx \int_{-\sin\frac{x}{2}}^{\sin x} f(x,y)dy$$

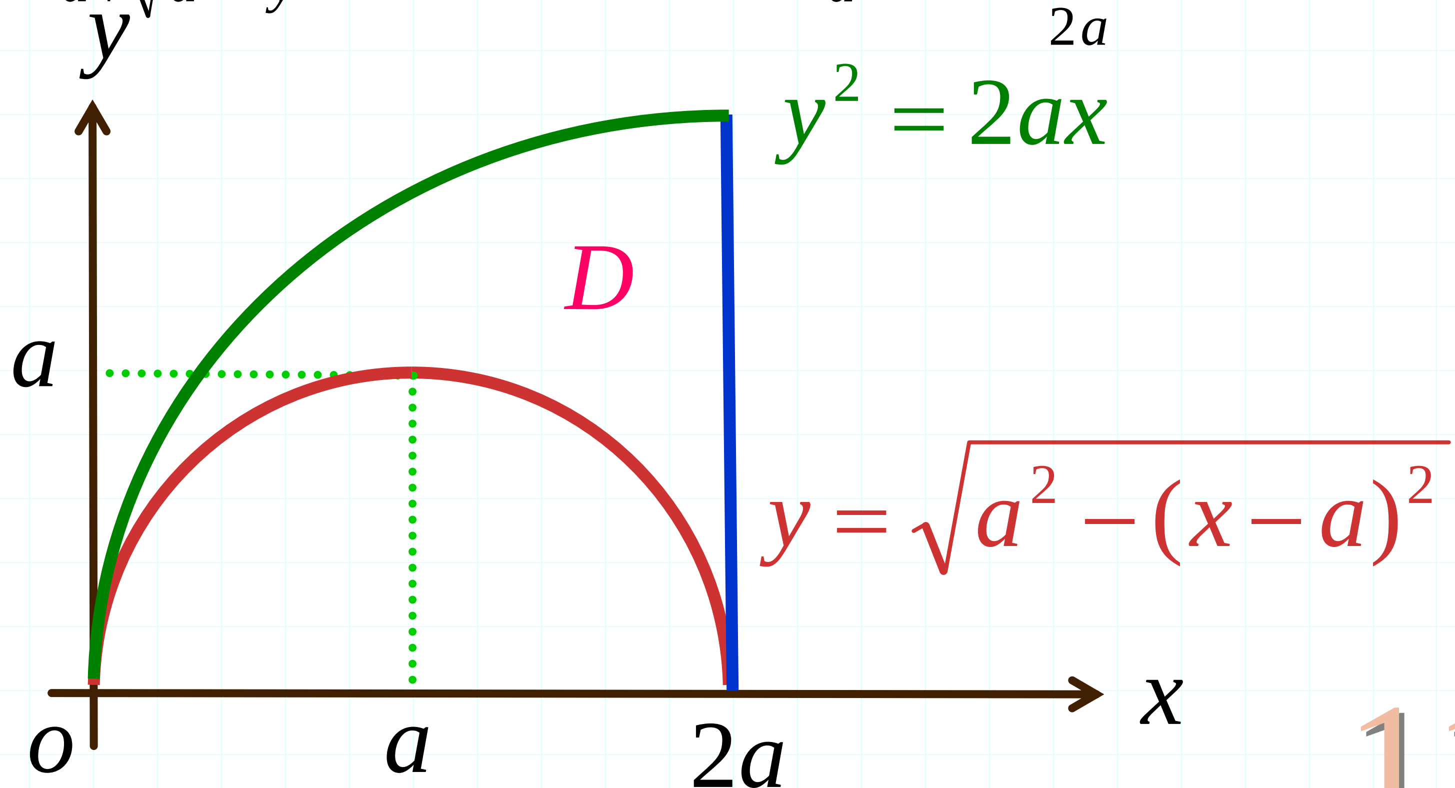
$$= \int_{-1}^{0} dy \int_{-2\arcsin y}^{\pi} f(x,y)dx + \int_0^1 dy \int_{\arcsin y}^{\pi - \arcsin y} f(x,y)dx$$



习题9～2：P366. 5(6)

$$\int_0^{2a}dx\int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}}f(x,y)dy=\int_0^{a}dy\int_{\frac{y^2}{2a}}^{a-\sqrt{a^2-y^2}}f(x,y)dx+$$

$$+\int_0^{a}dy\int_{a+\sqrt{a^2-y^2}}^{2a}f(x,y)dx+\int_a^{2a}dy\int_{\frac{y^2}{2a}}^{2a}f(x,y)dx$$



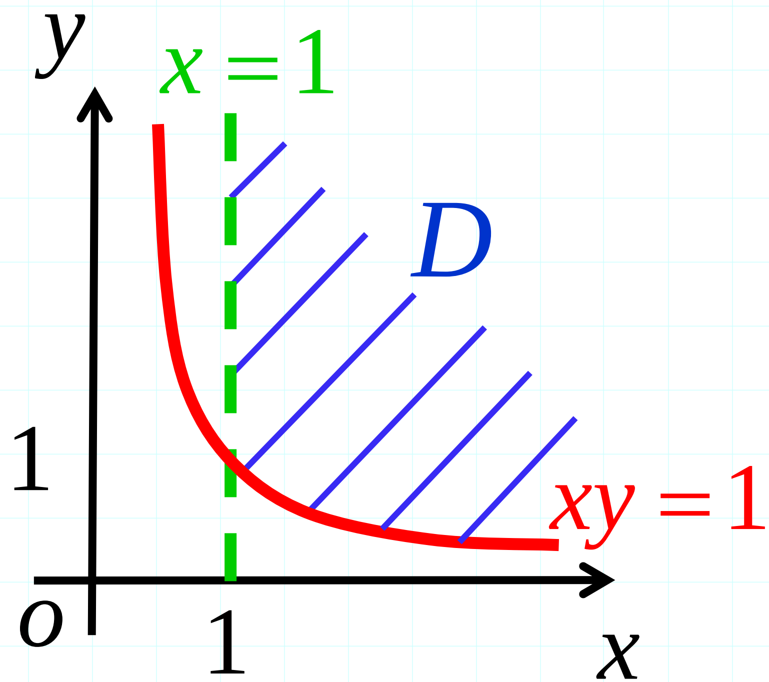
习题9～2：P367. 16（1）

$$\iint_D \frac{d\sigma}{x^p \cdot y^q} \qquad D=\{(x,y)\mid xy\ge 1, x\ge 1\}$$

$$=\int_1^{+\infty}\frac{dx}{x^p}\int_{\frac{1}{x}}^{+\infty}\frac{dy}{y^q}$$

$$=\int_1^{+\infty}\frac{1}{x^p}\left[\frac{-1}{(q-1)y^{q-1}}\right]_{\frac{1}{x}}^{+\infty}dx$$

$$=\int_1^{+\infty}\frac{1}{q-1}x^{q-1-p}dx \quad (q>1)$$



$$(p>q>1)$$

$$=\frac{1}{(q-1)(q-p)}x^{q-p}\Big|_1^{+\infty}=\frac{1}{(q-1)(q-p)}$$

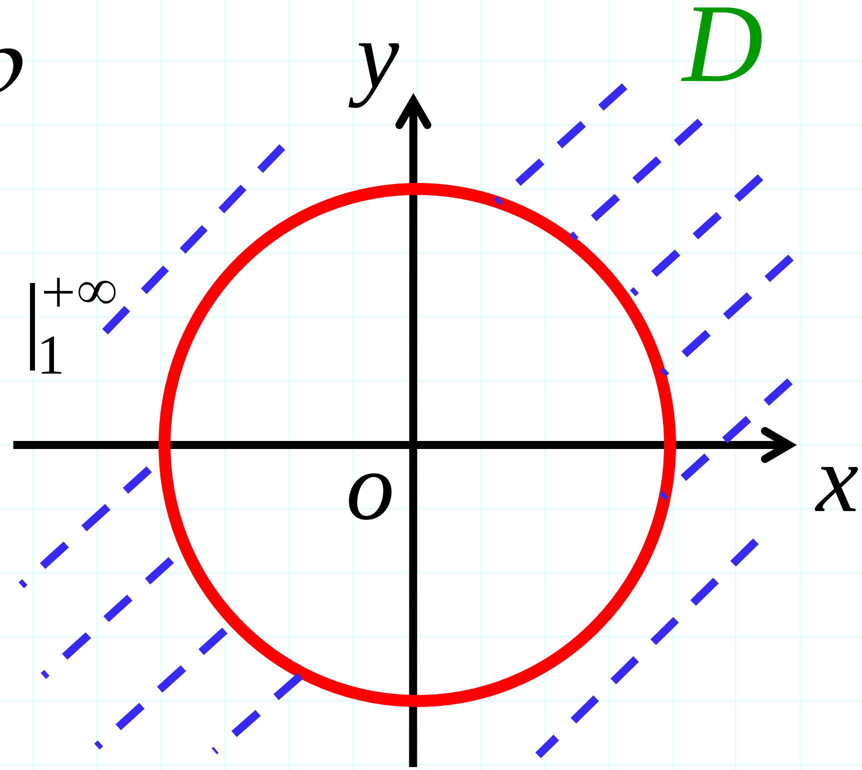
习题9～2：P367.16(2)

$$\iint\limits_D \frac{d\sigma}{(x^2+y^2)^p} \qquad D=\{(x,y)\mid x^2+y^2\geq 1\}$$

$$=\int_0^{2\pi} d\theta \int_1^{+\infty} \frac{1}{\rho^{2p}}\cdot\rho\cdot d\rho$$

$$=2\pi\cdot\frac{1}{-2p+2}\cdot\rho^{-2p+2}\Big|_1^{+\infty}$$

$$=\frac{\pi}{p-1} \qquad (p>1)$$



总习题九：P367. 1（1）

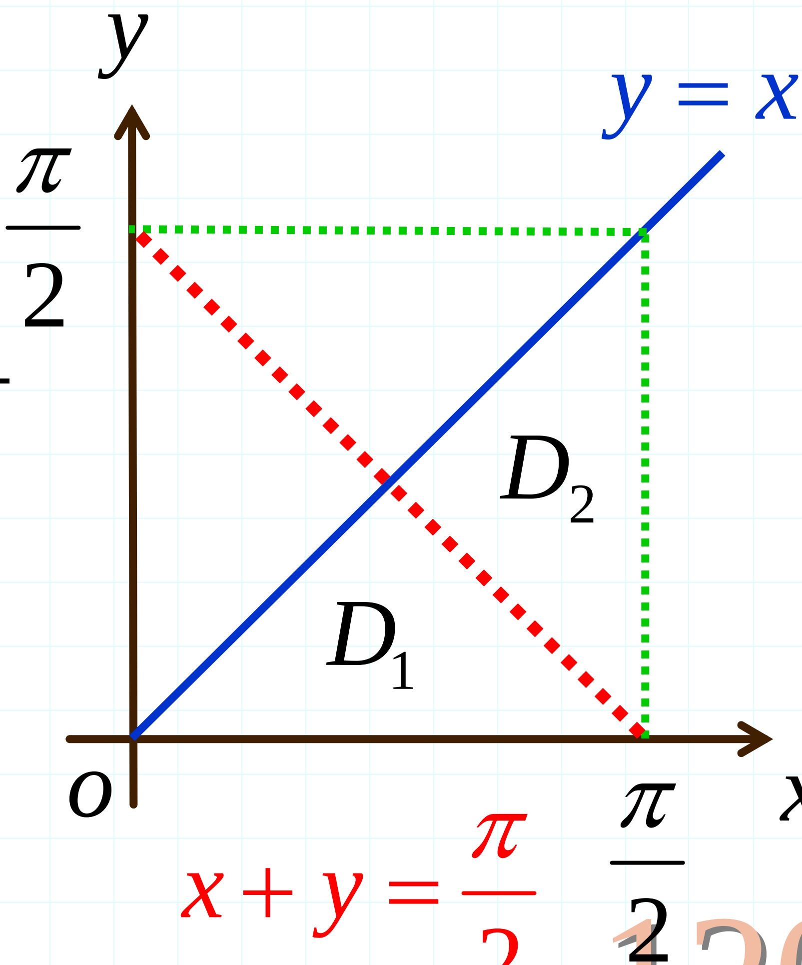
$$\iint_D |\cos(x+y)|d\sigma \quad y=x, y=0, x=\frac{\pi}{2}$$ 所围区域

$$=\int_0^{\frac{\pi}{2}}dx\int_0^x|\cos(x+y)|dy$$

$$=\int_0^{\frac{\pi}{4}}dy\int_y^{\frac{\pi}{2}-y}\cos(x+y)dx-$$

$$-\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}dx\int_{\frac{\pi}{2}-x}^{x}\cos(x+y)dy$$

$$=\frac{\pi}{2}-1$$



总习题九：P368．6　　（3－1）

$$\iint_D f(x-y)d\sigma = \int_{-A}^{A} f(t)(A-|t|)dt$$

$$D = \{(x,y) \mid |x| \le \frac{A}{2}, |y| \le \frac{A}{2}\}$$

证： $x-y=t \Rightarrow y = x-t$ $\begin{cases} x = x(x,t) = x \\ y = y(x,t) = x-t \end{cases}$

$$J(x,t) = \begin{vmatrix} \dfrac{\partial x}{\partial x} & \dfrac{\partial x}{\partial t} \\ \dfrac{\partial y}{\partial x} & \dfrac{\partial y}{\partial t} \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} = -1$$

$$dxdy = |J|dxdt = dxdt$$

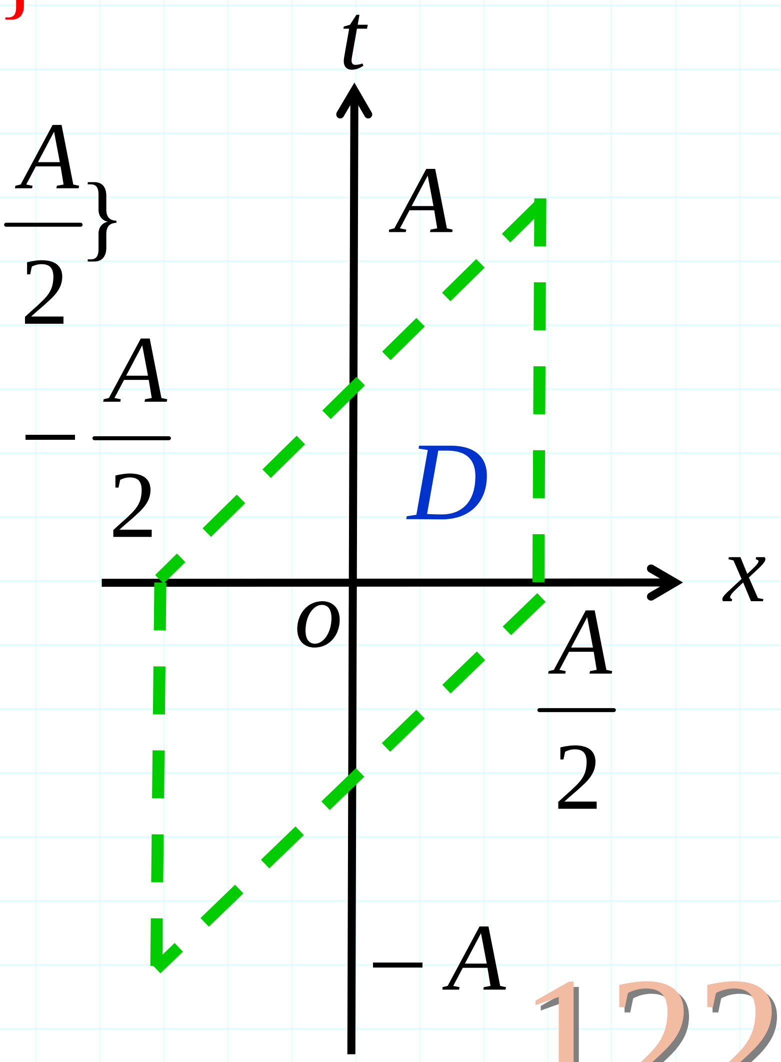
总习题九：P368．6（续） （3－2）

$$D=\{(x,y)\mid |x|\le \frac{A}{2}, |y|\le \frac{A}{2}\}$$

$$D=\{(x,t)\mid |x|\le \frac{A}{2}, |x-t|\le \frac{A}{2}\}$$

$$\left\{\begin{array}{c} -A\le t\le 0 \\ -\frac{A}{2}\le x\le t+\frac{A}{2} \end{array}\right\}\cup$$

$$\cup\left\{\begin{array}{c} 0\le t\le A \\ t-\frac{A}{2}\le x\le \frac{A}{2} \end{array}\right\}$$



总习题九：P368．6（续） （3－3）

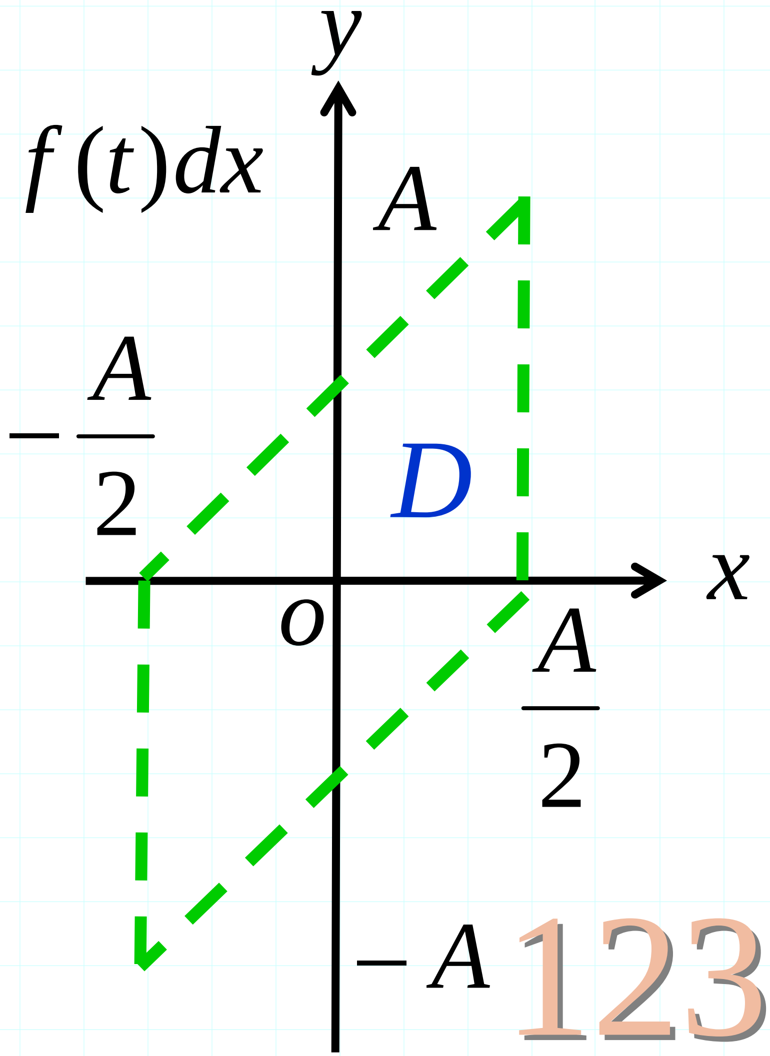
$$\iint_D f(x-y)d\sigma=\int_{-A}^{A}f(t)(A-|t|)dt$$

$$=\int_{-A}^{0}dt\int_{-\frac{A}{2}}^{t+\frac{A}{2}}f(t)dx+\int_{0}^{A}dt\int_{t-\frac{A}{2}}^{\frac{A}{2}}f(t)dx$$

$$=\int_{-A}^{0}f(t)(A+t)dt+$$

$$+\int_{0}^{A}f(t)(A-t)dt$$

$$=\int_{-A}^{A}f(t)(A-|t|)dt$$



习题１０～１：Ｐ３９２．５

$$\frac{d^2y}{dx^2}+9y=0 \qquad y=\cos\omega t \text{ 是解}$$

$$y'=-\sin\omega t\cdot\omega \qquad y''=-\cos\omega t\cdot\omega^2$$

$$-\cos\omega t\cdot\omega^2+9\cos\omega t=0$$

$$\cos\omega t\cdot(9-\omega^2)=0$$

$$\omega=\pm3$$

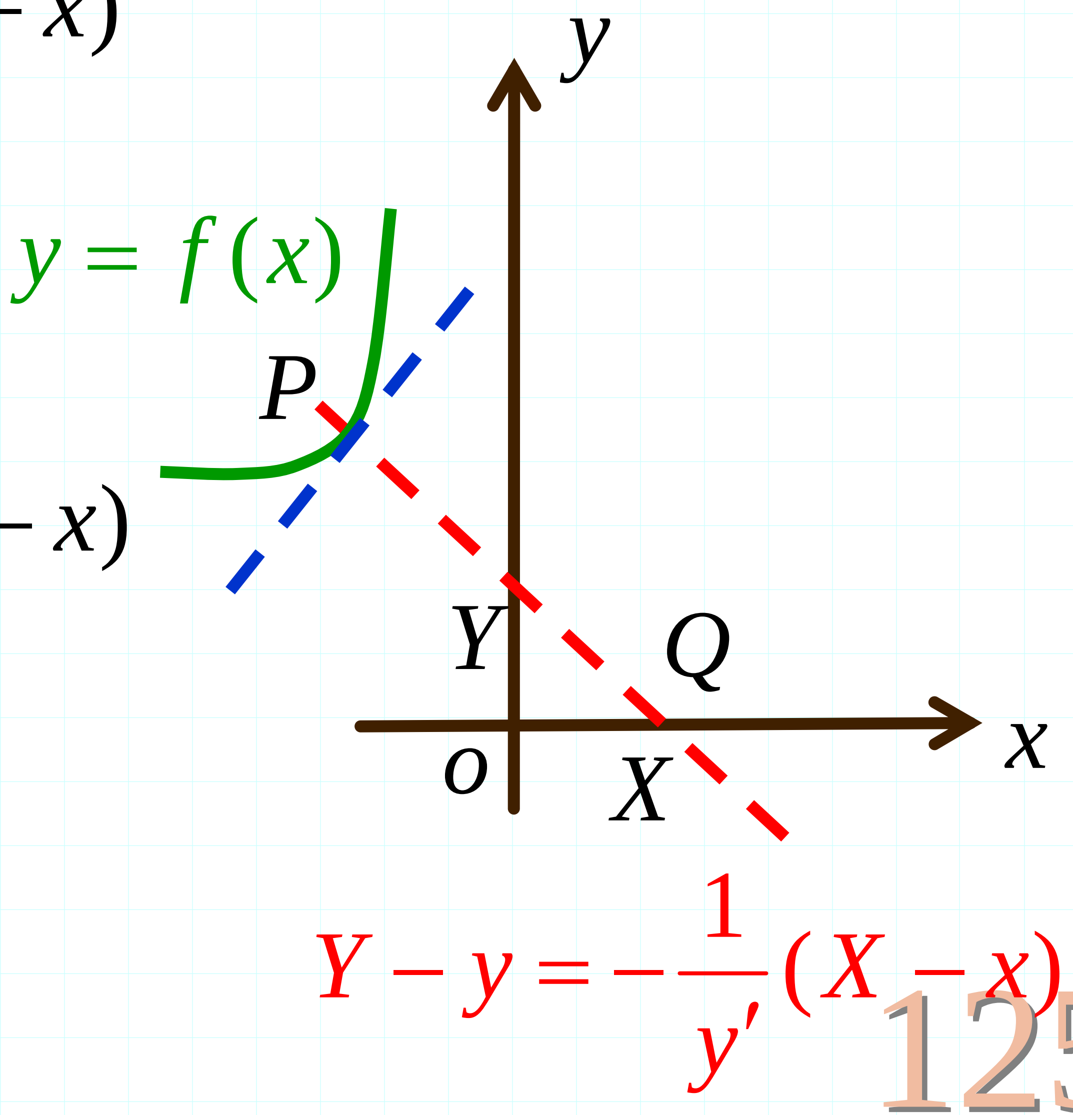
习题１０～１：Ｐ３９２．９

$$Y-y=-\frac{1}{y'}(X-x)$$

$$X=0, Y=\frac{y}{2}$$

$$\frac{y}{2}-y=-\frac{1}{y'}\cdot(0-x)$$

$$y\cdot y'+2x=0$$



习题１０～１：Ｐ３９２．１０

$$x = x(P)$$

$$R = x \cdot P = x(P) \cdot P$$

$$R \equiv C \;\Rightarrow\; x(P) \cdot P \equiv C$$

$$\Rightarrow \frac{d}{dP}(x(P) \cdot P) = 0 \qquad x(P) + P\frac{dx}{dP} = 0$$

$$\frac{Ex}{EP} = \frac{P}{x} \cdot \frac{dx}{dP} = -1$$

习题１０～２：Ｐ４０３．２（６）

$$(1+2e^{\frac{x}{y}})dx+2e^{\frac{x}{y}}(1-\frac{x}{y})dy=0$$

$$\frac{dx}{dy}=\frac{2e^{\frac{x}{y}}(\frac{x}{y}-1)}{1+2e^{\frac{x}{y}}} \qquad \frac{x}{y}=u \qquad y\frac{du}{dy}=\frac{-u-2e^{u}}{1+2e^{u}}$$

$$-\frac{1+2e^{u}}{u+2e^{u}}du=\frac{dy}{y} \qquad -\ln\left|u+2e^{u}\right|=\ln|y|+C_1$$

$$(u+2e^{u})\cdot y=C \qquad 2y\cdot e^{\frac{x}{y}}+x=C$$

习题１０～２：Ｐ４０３．４（８）

$$\frac{dy}{dx}+3y=8 \quad y|_{x=0}=2$$

$$P(x)=3, Q(x)=8$$

$$y=e^{-\int P(x)dx}\left(\int Q(x)e^{\int P(x)dx}dx+C\right)$$

$$=e^{-3x}\left(\int 8e^{3x}dx+C\right)$$

$$=e^{-3x}\left(\frac{8}{3}e^{3x}+C\right)=\frac{8}{3}+C\cdot e^{-3x}=\frac{8}{3}-\frac{2}{3}e^{-3x}$$

$$y|_{x=0}=2 \Rightarrow C=-\frac{2}{3}$$

习题１０～２：Ｐ４０３．５

$$\frac{dR}{dt} = k \cdot R \Rightarrow R = C \cdot e^{k \cdot t}$$

$$R_0 = C \cdot e^{k \cdot 0} \Rightarrow C = R_0 \Rightarrow R = R_0 \cdot e^{k \cdot t}$$

$$\frac{R_0}{2} = R_0 \cdot e^{k \times 1600} \Rightarrow k = -0.000433$$

$$R = R_0 \cdot e^{-0.000433 \cdot t}$$

习题１０～２：Ｐ４０３．６

$$y' = 2x + y \quad y(0) = 0$$

$$y = -2x - 2 + C \cdot e^{x}$$

$$y(0) = 0 \;\Rightarrow 0 = 0 - 2 + C \cdot e^{0} \;\Rightarrow C = 2$$

$$y = -2x - 2 + 2e^{x} = 2(e^{x} - x + 1)$$

习题１０～２：Ｐ４０３．７

$$\int_0^x [2f(t)-1]dt = f(x)-1$$

两边求导　$2f(x)-1 = f'(x)$

$$y'-2y=-1 \Rightarrow y=\frac{1}{2}+C\cdot e^{2x}$$

$$f(0)-1=0 \quad \Rightarrow C=\frac{1}{2}$$

$$y=\frac{1}{2}(1+e^{2x})$$

习题１０～３：Ｐ４１０．２

$$\frac{P}{Q}\cdot\frac{dQ}{dP}=-3P^2$$

$$\frac{dQ}{dP}+3P^2\cdot Q=0$$

$$Q=C\cdot e^{-3P}$$

$$\Rightarrow Q=e^{-3P}$$

$$Q\,|_{P=0}=1$$

习题１０～３：Ｐ４１０．４

$$5\% \cdot W_0 = 1{,}2000 \Rightarrow W_0 = 24{,}0000$$

$$\frac{dW}{dt} = 5\% \cdot W - 1{,}2000$$

$$W = 240{,}000 + C \cdot e^{0.05 \cdot t}$$

$$0 = 240{,}000 + C \cdot e^{0.05 \times 20} \Rightarrow C = -24{,}0000 e^{-1}$$

$$\frac{dB}{dt} = 5\% \cdot B - 1{,}2000$$

$$B = 24{,}0000 + C \cdot e^{0.05 \cdot t}$$

$$B_0 = 24{,}0000 - 24{,}0000 e^{-1}$$ 余额为零

习题１０～３：Ｐ４１０．５

$$\frac{dy}{dt} = k \cdot y \cdot (1000 - y) \quad (k > 0)$$

$$y|_{t=0} = 100 \quad y|_{t=3} = 250$$

$$\frac{y}{1000 - y} = C \cdot e^{1000 \cdot k \cdot t} \quad y|_{t=0} = 100 \Rightarrow C = \frac{1}{9}$$

$$y|_{t=3} = 250 \Rightarrow k = \frac{\ln 3}{3000}$$

$$y = \frac{1000 \cdot 3^{\frac{t}{3}}}{9 + 3^{\frac{t}{3}}} \quad y|_{t=6} = 500$$

总习题十：P453. 5

$$\int_0^x f(t)dt = x + \int_0^x tf(x-t)dt$$

$$\int_0^x tf(x-t)dt \overset{\substack{x-t=u,t=x-u,dt=-du \\ t:0\to x;u:x\to 0}}{=\!=\!=\!=\!=\!=\!=\!=\!=\!=} \int_x^0 (x-u)f(u)(-du)$$

$$= \int_0^x (x-u)f(u)du = x\int_0^x f(u)du - \int_0^x uf(u)du$$

两边求导 $f(x) = 1 + \int_0^x f(u)du + xf(x) - xf(x)$

$$\begin{cases} f(0) = 1 \\ f'(x) = f(x) \end{cases} \Rightarrow y = e^x$$

总习题十： P 4 5 3． 6

$x\cdot y'+P(x)y=x$ $y=e^{x}$ 是解 $y'+\frac{P(x)}{x}\cdot y=1$

$$e^{x}+\frac{P(x)}{x}\cdot e^{x}=1\Rightarrow\frac{P(x)}{x}=e^{-x}-1$$

$$y'+(e^{-x}-1)\cdot y=1$$

$$y=e^{-\int P(x)dx}\left(\int Q(x)e^{\int P(x)dx}dx+C\right)$$

$$=e^{-\int(e^{-x}-1)dx}\left(\int 1\cdot e^{\int(e^{-x}-1)dx}dx+C\right)$$

$$=e^{x+e^{-x}}\left(e^{e^{-x}}+C\right)\qquad y=e^{x}-e^{x+e^{-x}-\frac{1}{2}}$$

$$y\big|_{x=\ln 2}=0\Rightarrow C=-e^{-\frac{1}{2}}$$

总习题十：P453. 7

$$y - y_0 = f'(x_0)\cdot(x - x_0)$$

$$y = f'(x_0)\cdot x - f'(x_0)\cdot x_0 + y_0$$

$$x = 0 \qquad -f'(x_0)\cdot x_0 + y_0 = x_0$$

即： $y'\cdot x - y = -x \Rightarrow y' - \frac{1}{x}\cdot y = -1$

$$y = e^{-\int(\frac{-1}{x})dx}\left(\int(-1)e^{\int(\frac{-1}{x})dx}dx + C\right)$$

$$= -x\cdot\ln x + Cx$$

$$y\big|_{x=1} = 1 \ \Rightarrow C = 1$$

$$y = x - x\cdot\ln x$$

习题１１～３：Ｐ４７７．１（１）

$$\sum_{n=1}^{\infty}(-1)^n\cdot\sqrt{\frac{n}{3n+1}}$$

$$u_n=\sqrt{\frac{n}{3n+1}}\xrightarrow{n\to\infty}\sqrt{\frac{1}{3}}\neq 0$$

发散

习题１１～３：Ｐ４７７．１（２）

$$\sum_{n=1}^{\infty}(-1)^n\cdot\sin\frac{1}{n}$$

$$\lim_{n\to\infty}u_n=\lim_{n\to\infty}\sin\frac{1}{n}=0$$

$$u_n-u_{n+1}=\sin\frac{1}{n}-\sin\frac{1}{n+1}$$

$$=2\cdot\cos\frac{\frac{1}{n}+\frac{1}{n+1}}{2}\cdot\sin\frac{\frac{1}{n}-\frac{1}{n+1}}{2}>0$$

收敛

习题１１～３：Ｐ４７７．２（１）

$$\sum_{n=1}^{\infty}(-1)^n\cdot\frac{1}{(2n-1)^2}$$

$$\frac{1}{(2n-1)^2}<\frac{1}{n^2}$$

$$\sum_{n=1}^{\infty}\frac{1}{n^2}<\infty\Rightarrow\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}<\infty$$

绝对收敛

习题11～3：P477.2（2）

$$\sum_{n=1}^{\infty}(-1)^n \cdot \frac{n}{2^n}$$

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty}\left|\frac{\dfrac{n+1}{2^{n+1}}}{\dfrac{n}{2^n}}\right| = \lim_{n\to\infty}\frac{n+1}{n}\cdot\frac{1}{2} = \frac{1}{2} < 1$$

绝对收敛

习题１１～３：Ｐ４７７．２（３）

$$\sum_{n=1}^{\infty}\frac{1}{n}\cdot\sin\frac{n\cdot\pi}{2}=\sum_{k=1}^{\infty}(-1)^{k-1}\cdot\frac{1}{2k-1}$$

$$\sum_{k=1}^{\infty}\frac{1}{2k-1}=\infty$$

$$\lim_{k\to\infty}u_k=\lim_{k\to\infty}\frac{1}{2k-1}=0$$

$$u_k-u_{k+1}=\frac{1}{2k-1}-\frac{1}{2k+1}>0$$

条件收敛

习题１１～３：Ｐ４７７．２（４）

$$\sum_{n=1}^{\infty}(-1)^n\cdot\left(1-\cos\frac{1}{n}\right)$$

$$\lim_{n\to\infty}\frac{1-\cos\frac{1}{n}}{\frac{1}{n^2}}\overset{\frac{1}{n}=t}{=\!=}\lim_{t\to0}\frac{1-\cos t}{t^2}\overset{\left(\frac{0}{0}\right)}{=\!=}\lim_{t\to0}\frac{\sin t}{2t}=\frac{1}{2}$$

$$\sum_{n=1}^{\infty}\frac{1}{n^2}<\infty\Rightarrow\sum_{n=1}^{\infty}\left(1-\cos\frac{1}{n}\right)<\infty$$

绝对收敛

习题１１～３：Ｐ４７７．２（５）

$$\sum_{n=1}^{\infty}(-1)^n \cdot \frac{n}{2n+1}$$

$$\lim_{n\to\infty} u_n = \lim_{n\to\infty}\frac{n}{2n+1} = \frac{1}{2} \neq 0$$

发散

习题１１～３：Ｐ４７７．２（６）

$$\sum_{n=1}^{\infty}(-1)^n\cdot\ln\left(\frac{n+1}{n}\right)$$

$$\lim_{n\to\infty}u_n=\lim_{n\to\infty}\ln\left(\frac{n+1}{n}\right)=0$$

$$\lim_{n\to\infty}\frac{\ln\left(\frac{n+1}{n}\right)}{\frac{1}{n}}$$

$$=\lim_{n\to\infty}n\cdot\ln\left(\frac{n+1}{n}\right)=e$$

$$u_n-u_{n+1}$$

$$=\ln\left(\frac{n+1}{n}\right)-\ln\left(\frac{n+2}{n+1}\right)$$

$$=\ln\left(1+\frac{1}{n}\right)-\ln\left(1+\frac{1}{n+1}\right)>0$$

条件收敛

习题１１～３：Ｐ４７８．２（７）

$$\sum_{n=1}^{\infty}(-1)^n\cdot\frac{x^{2n-1}}{(2n-1)!}$$

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right|=\lim_{n\to\infty}\left|\frac{\dfrac{x^{2n+1}}{(2n+1)!}}{\dfrac{x^{2n-1}}{(2n-1)!}}\right|=\lim_{n\to\infty}\frac{x^2}{(2n+1)\cdot(2n)}=0$$

对一切 x 绝对收敛

习题11～3：P478．3

$$|a \cdot b| \le \frac{1}{2}(a^2 + b^2)$$

$$\frac{|a_n|}{\sqrt{n^\alpha + \lambda}} \le \frac{|a_n|}{n^{\frac{\alpha}{2}}} \le \frac{1}{2}(a_n^2 + \frac{1}{n^\alpha}) \qquad (\lambda > 0)$$

$$\sum_{n=1}^{\infty} a_n^2 < +\infty \qquad \sum_{n=1}^{\infty} \frac{1}{n^\alpha} < +\infty \quad (\alpha > 1)$$

$$\sum_{n=1}^{\infty} (-1)^n \frac{|a_n|}{\sqrt{n^\alpha + \lambda}}$$ 绝对收敛